Math 1172 Catch-up Exam #1

Question 9. a) Please find the derivative of $f(x) = x \tan^{-1}(3x + 2)$

$$x \cdot \frac{1}{1+(3x+2)^2} \cdot 3 + \tan^{-1}(3x+2) \cdot 1$$

b) In each part, evaluate the inverse trig function by hand. If the answer is undefined, say "undefined". If it's defined, but impossible to do by hand, say "impossible".

i) $\sin^{-1} 1$ $\pi/2$

ii) $\tan^{-1} \frac{1}{2}$ impossible

iii) $\cos^{-1} + \frac{\sqrt{3}}{2}$ $\pi/6$

iv) $\arcsin \frac{1}{\sqrt{2}}$ $\pi/4$

Question 10. Please find the limit:

$$\lim_{x\to 0} \frac{x3^x}{3^x-1}$$

$$\lim_{x\to 0} \frac{x3^x}{3^x-1} = \lim_{x\to 0} \frac{x\cdot 3^x \ln 3 + 3^x}{3^x \ln 3}$$

$$= \lim_{x\to 0} \frac{x\ln 3 + 1}{\ln 3}$$

$$\boxed{\frac{1}{\ln 3}}$$

Question 11. Please find the antiderivative of $f(x) = \dfrac{\ln x}{x^3}$.

$$\int \frac{\ln x}{x^3}\,dx \qquad u = \ln x \quad du = \tfrac{1}{x}\,dx \qquad dv = x^{-3} \quad v = -\tfrac{1}{2}x^{-2}$$

$$\ln x\left(-\tfrac{1}{2}x^{-2}\right) - \int -\tfrac{1}{2}x^{-2}\cdot\tfrac{1}{x}\,dx$$

$$-\tfrac{1}{2}x^{-2}\ln x + \tfrac{1}{2}\int x^{-3}\,dx$$

$$-\tfrac{1}{2}x^{-2}\ln x + \tfrac{1}{2}\cdot\tfrac{1}{-2}x^{-2} + C$$

Old Ones

Do only the ones you want. Write your answers on the blank paper.

If there are a,b, etc parts, you must do all of them! You are re-attempting the entire question, not just one part of it.

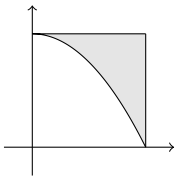
Question 1. a) Please find $f'(\pi/3)$ for $f(x) = (x \sin x)^2$ and simplify any trig functions in the answer.

b) Please find: $\int x(x^2-1)^2\,dx$

Question 2. Please do the integral: $\int_0^2 x^4(7-x^5)^3\,dx$

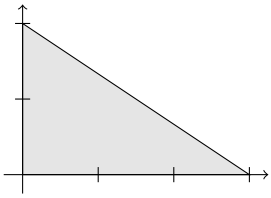
Question 3. Please find the area bounded between $x = y^2 + 4$ and $y = \frac{1}{4}x - 1$.

Question 4. Please use washers to find the volume obtained by revolving this shape around the line $y = 3$:



The curve is $y = x^2 + 1$.

Question 5. Please use shells to find the volume obtained by revolving this shape around the line $x = 3$:



Each mark on the axes is 1 unit.

Question 6. a) Please find $f'(0)$ for $f(x) = (\ln(\sqrt{x-1}))^3$ and simplify as much as you can.

b) Please find the integral: $\int \frac{1+3x-2x^2}{x^2}\,dx$

Question 7. a) Please find the derivative of $f(x) = (4x + e^{2x})^2$

b) Please find the antiderivative of xe^{7-x^2}

Question 8. a) Please find each logarithm by hand. If the answer is undefined, say "undefined". If it's defined, but impossible to do by hand, say "impossible".

i) $\log_2 8$

ii) $\log_5 0$

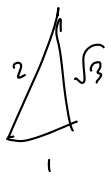
iii) $\log_4 \frac{1}{2}$

iv) $\log_3 81$

b) Please find the derivative of $f(x) = 4 \cdot 5^{x^2} + x^5$

Question 1. a) Please find $f'(\pi/3)$ for $f(x) = (x \sin x)^2$ and simplify any trig functions in the answer.

b) Please find: $\int x(x^2-1)^2\,dx$



a $\quad f'(x) = 2(x\sin x)(x\cdot\cos x + \sin x)$

$f'(\pi/3) = 2\left(\frac{\pi}{3}\sin\pi/3\right)\left(\pi/3\cos\pi/3 + \sin\pi/3\right)$

$= 2\cdot\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$

b $\quad \int x(x^2-1)^2\,dx = \int x(x^4-2x^2+1)\,dx$

$= \int x^5 - 2x^3 + x\,dx = \frac{1}{6}x^6 - \frac{2}{4}x^4 + \frac{1}{2}x^2 + C$

Question 2. Please do the integral: $\int_0^2 x^4(7-x^5)^3\,dx$

$u = 7-x^5$

$du = -5x^4\,dx$

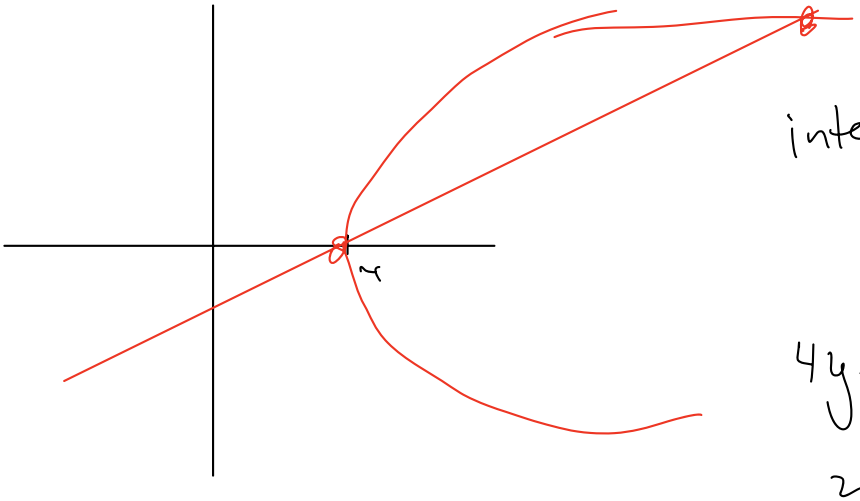
$-\frac{1}{5}du = x^4\,dx$

$\int u^3 \cdot x^4\,dx = \int u^3 \cdot \frac{-1}{5}\,du$

$= -\frac{1}{5}\int u^3\,du = \frac{-1}{5}\cdot\frac{1}{4}u^4$

$= \frac{-1}{20}(7-x^5)^4\Big|_0^2 = \frac{-1}{20}(7-2^5)^4 - \frac{-1}{20}(7-0)^4$

Question 3. Please find the area bounded between $x = y^2 + 4$ and $y = \frac{1}{4}x - 1$.



$x = 4y + 4$

intersections:

$x = y^2 + 4$

$4y + 4 = y^2 + 4$

$y^2 - 4y = 0$

$y(y - 4) = 0$

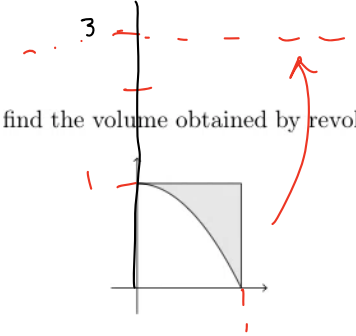
$y = 0, 4$

$$\int_0^4 (4y + 4) - (y^2 + 4)\, dy = \int_0^4 4y - y^2\, dy$$

$$= 2 \cdot y^2 - \frac{1}{3}y^3 \Big|_0^4$$

$$= 2 \cdot 4^2 - \frac{1}{3} \cdot 4^3$$

Question 4. Please use washers to find the volume obtained by revolving this shape around the line $y = 3$:



inner $= 3 - 1 = 2$

outer $= 3 - (1 - x^2)$

$= 2 + x^2$

The curve is $y = \cancel{x^2 + 1.}$ $1 - x^2$

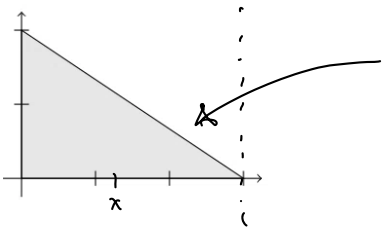
$$\pi \int_0^1 (2 + x^2)^2 - 2^2 \, dx$$

$$= \pi \int_0^1 \cancel{4} + 4x^2 + x^4 - \cancel{4} \, dx$$

$$= \pi \int_0^1 4x^2 + x^4 \, dx = \pi \left(\frac{4}{3} x^3 + \frac{1}{5} x^5 \right) \Big|_0^1$$

$$= \pi \left(\frac{4}{3} + \frac{1}{5} \right)$$

Question 5. Please use shells to find the volume obtained by revolving this shape around the line $x = 3$:



$y = \frac{-2}{3}x + 2$

Each mark on the axes is 1 unit.

height $= \frac{-2}{3}x + 2$

dist $= 3 - x$

$$2\pi \int_0^3 (3-x)\left(-\frac{2}{3}x+2\right)\,dx$$

$$2\pi \int_0^3 -2x + \frac{2}{3}x^2 - 2x + 6\,dx$$

$$2\pi \int_0^3 -4x + \frac{2}{3}x^2 + 6\,dx$$

$$= 2\pi\left(-2x^2 + \frac{2}{9}x^3 + 6x\right)\Big|_0^3$$

$$= 2\pi\left(-2\cdot 3^2 + \frac{2}{9}\cdot 3^3 + 6\cdot 3\right)$$

f'(2)

Question 6. a) Please find $f'(4)$ for $f(x) = (\ln(\sqrt{x-1}))^3$ and simplify as much as you can.

b) Please find the integral: $\int \frac{1+3x-2x^2}{x^2}\,dx$

a) $f'(x) = 3(\ln\sqrt{x-1})^2 \cdot \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2}(x-1)^{-1/2}$

$f'(2) = 3(\ln\sqrt{1})^2 \cdot \frac{1}{\sqrt{1}} \cdot \frac{1}{2}(1)^{-1/2}$

$\uparrow$

0

$= 0$

b) $\int \frac{1+3x-2x^2}{x^2}\,dx = \int x^{-2} + 3x^{-1} - 2\,dx$

$= -x^{-1} + 3\ln|x| - 2x + C$

Question 7. a) Please find the derivative of $f(x) = (4x + e^{2x})^2$

b) Please find the antiderivative of xe^{7-x^2}

a) $2(4x+e^{2x})(4+e^{2x}\cdot 2)$

b) $\int xe^{7-x^2}\,dx$ $\quad u = 7-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \int e^u x\,dx$

$$= -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{7-x^2} + C$$

Question 8. a) Please find each logarithm by hand. If the answer is undefined, say "undefined". If it's defined, but impossible to do by hand, say "impossible".

i) $\log_2 8$ 3

ii) $\log_5 0$ undefined

iii) $\log_4 \frac{1}{2}$ $-1/2$

iv) $\log_3 81$ 4

b) Please find the derivative of $f(x) = 4 \cdot 5^{x^2} + x^5$

$$4 \cdot 5^{x^2} \ln 5 \cdot 2x + 5x^4$$