Math 1172 Catch-up Exam #1

Question 9. Please do the integral, and simplify all inverse trig functions in your answer:

$$\int_0^{1/4} \frac{1}{\sqrt{1-4x^2}}\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\int \frac{1}{\sqrt{1-(2x)^2}}\,dx$$

$$= \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2}du = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{1}{2}\sin^{-1}u = \frac{1}{2}\sin^{-1}(2x)\Big|_0^{1/4}$$

$$= \frac{1}{2}\sin^{-1}(1/2) - \frac{1}{2}\sin^{-1}(0)$$

$$= \frac{1}{2}\cdot \pi/6 - \frac{1}{2}\cdot 0$$

Question 10. Please find the limit:

~~$\lim_{x \to 0} x \ln |x|$~~ $\lim_{x \to \infty} \dfrac{(\ln x)^2}{x^2}$

$$\lim_{x \to \infty} \frac{(\ln x)^2}{x^2} \overset{H}{=} \lim_{x \to \infty} \frac{2 \ln x \cdot \frac{1}{x}}{2x} = \lim_{x \to \infty} \frac{\ln x}{x^2} \overset{H}{=} \lim_{x \to \infty} \frac{1/x}{2x} = \lim_{x \to \infty} \frac{1}{2x^2} = \frac{1}{\infty} = 0$$

Question 11. Please do the integral:

$$\int_0^2 (5x-1)e^{3x}\,dx$$

$u = 5x-1 \qquad du = 5\,dx$

$dv = e^{3x}\,dx \qquad v = \frac{1}{3}e^{3x}$

$$uv - \int v\,du = (5x-1)\cdot\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\cdot 5\,dx$$

$$= (5x-1)\frac{1}{3}e^{3x} - \frac{5}{3}\int e^{3x}\,dx$$

$$= \cdots - \frac{5}{3}\cdot\frac{1}{3}e^{3x}\Big|_0^2$$

$$= (5\cdot 2-1)\frac{1}{3}e^{6} - \frac{5}{9}e^{6} - \left((5\cdot 0-1)\cdot\frac{1}{3}e^{0} - \frac{5}{9}e^{0}\right)$$

Old Ones

Do only the ones you want. Write your answers on the blank paper.

If there are a,b, etc parts, you must do all of them! You are re-attempting the entire question, not just one part of it.

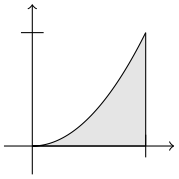
Question 1. a) Please find the derivative of $\dfrac{x\cos x}{x^2+1}$

b) Please find: $\displaystyle\int_{\pi/3}^{\pi/2} (\sin x - 3\cos x)\,dx$ and simplify all trig functions in the answer.

Question 2. Please do the integral: $\displaystyle\int x(x+1)^{10}\,dx$

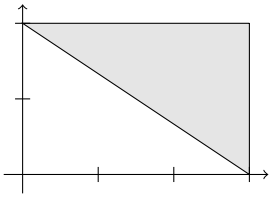
Question 3. Please find the area bounded between the curves $y = x^2 - 3x - 4$ and $x = y + 4$.

Question 4. Please use washers to find the volume obtained by revolving this shape around the line $x = 3$:



The curve is $y = x^2$, and the marks on the axes are 1 unit.

Question 5. Please use shells to find the volume obtained by revolving this shape around the line $x = -1$:



The marks on the axes are 1 unit.

Question 6. a) Please solve for x in:

$$3 \cdot 2^{4x+1} = 9$$

b) Please do the integral: $\displaystyle\int \frac{x}{3-2x^2}\,dx$

Question 7. a) Please find the derivative of $(3x^2 - \sin x)e^{\sqrt{x}}$

b) Please find the antiderivative of $e^x \cos(1 + e^x)$

Question 8. a) Please find the integral $\displaystyle\int (1 + 3^x)^2\,dx$

b) Please find $f'(0)$ for $f(x) = 3 \cdot 4^{x^2+1}$ and simplify as much as you can.

Question 1. a) Please find the derivative of $\dfrac{x\cos x}{x^2+1}$

b) Please find: $\displaystyle\int_{\pi/3}^{\pi/2}(\sin x - 3\cos x)\,dx$ and simplify all trig functions in the answer.

a) $$\frac{(x^2+1)\cdot(x\cdot -\sin x + \cos x\cdot 1) - x\cos x\cdot 2x}{(x^2+1)^2}$$

b) $$-\cos x - 3\sin x \Big|_{\pi/3}^{\pi/2}$$

$$= -\cos\pi/2 - 3\sin\pi/2 - (-\cos\pi/3 - 3\sin\pi/3)$$

$$= -0 - 3\cdot 1 - (-1/2 - 3\cdot\sqrt{3}/2)$$

Question 2. Please do the integral: $\displaystyle\int x(x+1)^{10}\,dx$

$u = x+1 \quad \rightarrow \quad x = u-1$

$du = dx$

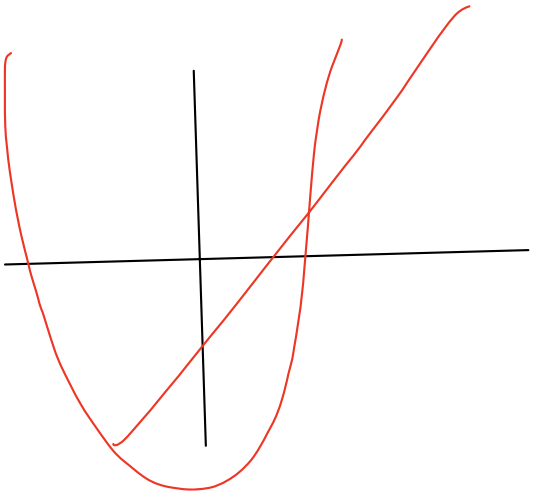
$$\int (u-1)u^{10}\,du = \int u^{11} - u^{10}\,du$$

$$= \frac{1}{12}u^{12} - \frac{1}{11}u^{11} + C$$

$$= \frac{1}{12}(x+1)^{12} - \frac{1}{11}(x+1)^{11} + C$$

Question 3. Please find the area bounded between the curves $y = x^2 - 3x - 4$ and $x = y + 4$.

$y = x - 4$



a parabola & a line

intersections:

$$x^2 - 3x - 4 = x - 4$$

$$x^2 - 4x = 0$$

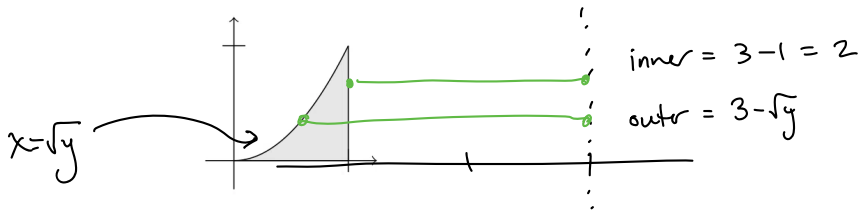
$$x(x-4) = 0$$

$$x = 0,\ x = 4$$

$$\int_0^4 (x-4) - (x^2-3x-4)\,dx = \int_0^4 x-4-x^2+3x+4\,dx$$

$$= \int_0^4 4x - x^2\,dx = 2x^2 - \frac{1}{3}x^3\Big|_0^4 = 2\cdot 4^2 - \frac{1}{3}\cdot 4^3 - (0-0)$$

Question 4. Please use washers to find the volume obtained by revolving this shape around the line $x = 3$:



The curve is $y = x^2$, and the marks on the axes are 1 unit.

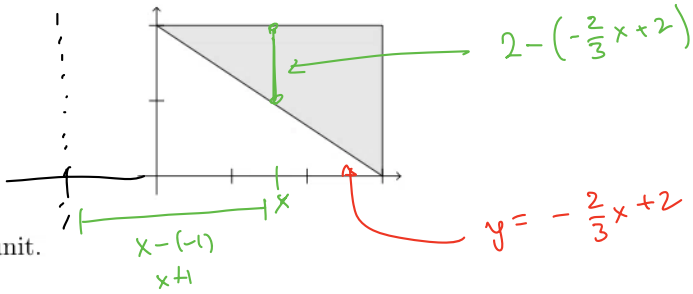
$$\pi \int_0^1 (3-\sqrt{y})^2 - 2^2\,dy$$

$$= \pi \int_0^1 9 - 6y^{1/2} + y - 4\,dy = \pi \int_0^1 5 - 6y^{1/2} + y\,dy$$

$$= \pi\left(5y - 6\cdot\frac{2}{3}y^{3/2} + \frac{1}{2}y^2\right)\Big|_0^1$$

$$= \pi\left(5 - 6\cdot\frac{2}{3} + \frac{1}{2}\right)$$

Question 5. Please use shells to find the volume obtained by revolving this shape around the line $x = -1$:



The marks on the axes are 1 unit.

$$2\pi \int_0^3 (x+1)\left(\frac{2}{3}x\right)dx = 2\pi \int_0^3 \frac{2}{3}x^2 + \frac{2}{3}x\,dx$$

$$= 2\pi\left(\frac{2}{9}x^3 + \frac{1}{3}x^2\right)\Big|_0^3$$

$$= 2\pi \left(\frac{2}{9} \cdot 3^3 + \frac{1}{3} \cdot 3^2 \right)$$

Question 6. a) Please solve for x in: $3 \cdot e^{4x+1} = 9$

b) Please do the integral: $\int \frac{x}{3-2x^2}\, dx$

a) $3 \cdot e^{4x+1} = 9$

$e^{4x+1} = 3$

$\ln 3 = 4x+1$

$x = \frac{\ln 3 - 1}{4}$

b) $\int \frac{x}{3-2x^2}\, dx$ $\quad u = 3-2x^2$, $du = -4x\, dx$, $-\frac{1}{4} du = x\, dx$

$= -\frac{1}{4} \int \frac{1}{u}\, du = -\frac{1}{4} \ln|u| + C$

$= -\frac{1}{4} \ln|3-2x^2| + C$

Question 7. a) Please find the derivative of $(3x^2 - \sin x)e^{\sqrt{x}}$

b) Please find the antiderivative of $e^x \cos(1+e^x)$

a) $(3x^2 - \sin x) \cdot e^{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2} + e^{\sqrt{x}} \cdot (6x - \cos x)$

b) $\int e^x \cos(1+e^x)\, dx$ $\quad u = 1+e^x$, $du = e^x\, dx$

$= \int \cos u\, du = \sin u + C$

$= \sin(1+e^x) + C$

Question 8. a) Please find the integral $\int 3^x(1+3^x)^{12}\,dx$

b) Please find $f'(0)$ for $f(x) = 3 \cdot 4^{x^2+1}$ and simplify as much as you can.

a) $\int 3^x (1+3^x)^{12}\,dx$

$u = 1+3^x$

$du = 3^x \ln 3\,dx$

$\frac{1}{\ln 3}\,du = 3^x\,dx$

$= \frac{1}{\ln 3}\int u^{12}\,du = \frac{1}{\ln 3}\cdot\frac{1}{13}u^{13} + C$

$= \frac{1}{\ln 3}\cdot\frac{1}{13}(1+3^x)^{13} + C$

b) $f'(x) = 3 \cdot 4^{x^2+1} \cdot \ln 4 \cdot 2x$

$f'(0) = 3 \cdot 4^1 \cdot \ln 4 \cdot 2 \cdot 0 = 0$