Math 1172 HW #6

7.2 #4, 7, 11, 18

7.2 #4

$$\int_0^{\pi/4} \sin^5 x\,dx = \int_0^{\pi/4} (\sin^2 x)^2 \sin x\,dx$$

$$= \int_0^{\pi/4} (1-\cos^2 x)^2 \sin x\,dx \qquad u = \cos x,\quad du = -\sin x\,dx$$

$$= -\int (1-u^2)^2\,du = -\int 1 - 2u^2 + u^4\,du$$

$$= -\left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right) = -\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x \Big|_0^{\pi/4}$$

$$= -\frac{1}{\sqrt{2}} - \frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{2}}\right)^5 - \left(-1 - \frac{2}{3} + \frac{1}{5}\right)$$

7.2 #7

$$\int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \frac{1}{2}(1+\cos 2x)\,dx$$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right)\Big|_0^{\pi/2} = \frac{1}{2}\left(\pi/2 + \frac{1}{2}\sin\pi\right) - \frac{1}{2}\left(0 + \frac{1}{2}\sin 0\right)$$

$$= \pi/4$$

7.2 #11

$$\int_0^{\pi/2} \sin^2 x \cos^2 x\,dx = \int_0^{\pi/2} \frac{1}{2}(1-\cos 2x)\cdot\frac{1}{2}(1+\cos 2x)\,dx$$

$$= \frac{1}{4}\int_0^{\pi/2} 1 - \cos^2 2x\,dx = \frac{1}{4}\int_0^{\pi/2} 1 - \frac{1}{2}(1+\cos 4x)\,dx$$

$$= \frac{1}{4}\int_0^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos 4x\,dx = \frac{1}{4}\left(\tfrac{1}{2}x - \frac{1}{8}\sin 4x\right)\Big|_0^{\pi/2}$$

$$= \frac{1}{4}\left(\pi/4 - \frac{1}{8}\sin 2\pi\right) - \frac{1}{4}\left(0 - \sin 0\right) = \pi/16$$

#18 $\int \tan^2 x \cos^3 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} \cos^3 x \, dx = \int \sin^2 x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int u^2 \, du = \frac{1}{3} u^3 + C = \frac{1}{3} \sin^3 x + C$