Math 5435

Homework #1

1.3 #4, 31

1.4 #10, 17

1.3 #4

$ax + by = f$
$cx + dy = g$

$$\left[\begin{array}{cc|c} a & b & f \\ c & d & g \end{array}\right]$$

we should start by subtracting $c/a \cdot R1$ from $R2$

$$\left[\begin{array}{cc|c} a & b & f \\ 0 & d - \frac{c}{a}b & g - \frac{c}{a}f \end{array}\right]$$

Second pivot is $d - \frac{c}{a}b$

Solve for y: $\left(d - \frac{c}{a}b\right) y = g - \frac{c}{a}f$

$$y = \frac{g - \frac{c}{a}f}{d - \frac{c}{a}b} = \frac{ag - cf}{ad - cb}$$

1.3 #31

$$\left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ a & a & 4 & b_2 \\ a & a & a & b_3 \end{array}\right]$$

no first pivot when $a = 0$

$$\rightarrow \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2 - b_1 \\ 0 & a-2 & a-3 & b_3 - b_1 \end{array}\right]$$

no second pivot when $a = 2$

$$\rightarrow \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2 - b_1 \\ 0 & 0 & a-4 & b_3 - b_1 - (b_2 - b_1) \end{array}\right]$$

no third pivot when $a = 4$

1.4 #10

a False!

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

A B AB

b False!

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

A B AB

c True! Row 1 in AB is obtained by products of (Row 1 in A) • (various cols in B)

If R1 of A = R3 of A, then these products will be the same, so R1 of AB will equal R3 of AB.

d False! $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ $B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A^2B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

not equal!

$AB = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

$$(AB)^2 = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

1.4 #17

$$(A+B)^2 = (A+B)(A+B)$$

we can distribute!

$$= A(A+B) + B(A+B)$$ and dist. again

$$= A^2 + AB + BA + B^2$$

It's also true that $(A+B)^2 = (A+B)(A+B)$

$$= (A+B)(B+A)$$

because matrix $+$ is commutative

It is not the same as $A^2 + 2AB + B^2$,

since $AB + BA$ may not equal $2AB$

since $AB \neq BA$.