

Math 5435 Homework #2

Section 1.5 # 4b, #11, 31b, 43

#4b

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 1 & 1 \\ 0 & 8/3 & 2/3 \\ 0 & 2/3 & 8/3 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 1 & 1 \\ 0 & 8/3 & 2/3 \\ 0 & 0 & 5/2 \end{bmatrix} \xrightarrow{8/3 - \frac{1}{4} \cdot \frac{2}{3}}$$

$$L = \begin{bmatrix} 1 & & \\ 1/3 & 1 & \\ 1/3 & 1/4 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 1 & 1 \\ & 8/3 & 2/3 \\ & & 5/2 \end{bmatrix}$$

#11

$$LU\vec{x} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ & 1 & 2 \\ & & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$L\vec{c} = \vec{b}: \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \vec{c} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{aligned} c_1 &= 2 \\ c_1 + c_2 &= 0 \rightarrow 2 + c_2 = 0 \rightarrow c_2 = -2 \\ c_1 + c_3 &= 2 \rightarrow 2 + c_3 = 2 \rightarrow c_3 = 0 \end{aligned}$$

$$\vec{c} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

$$U\vec{x} = \vec{c}: \begin{bmatrix} 2 & 4 & 4 \\ & 1 & 2 \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

$$z = 0$$

$$y + 2z = -2 \quad y = -2$$

$$2x + 4y + 4z = 2$$

$$2x - 8 + 0 = 2$$

$$2x = 10$$

$$x = 5$$

$$\vec{x} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$$

#316

$$\begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix} \rightarrow \begin{bmatrix} a & a & 0 \\ 0 & b & b \\ 0 & b & b+c \end{bmatrix} \rightarrow \begin{bmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \quad U = \begin{bmatrix} a & a & 0 \\ & b & b \\ & & c \end{bmatrix}$$

$$\text{or } A = \underset{L}{\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}} \underset{D}{\begin{bmatrix} a & & \\ & b & \\ & & c \end{bmatrix}} \underset{U}{\begin{bmatrix} 1 & 1 & \\ & 1 & \\ & & 1 \end{bmatrix}}$$

#43

$$A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$$

To get upper triangular, we need to permute rows like $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$,

$$\begin{pmatrix} 0 & 0 \\ & 0 \\ & & 0 \end{pmatrix}$$

so we do this to I to obtain

$$P = \begin{bmatrix} & 1 & \\ & & 1 \\ 1 & & \end{bmatrix}$$

To get lower triangular, we swap rows $2 \leftrightarrow 3$

by $P_1 = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$ to obtain

$$P_1 A = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$$

Now swap cols $1 \leftrightarrow 3$, which is a

right - mult. by $P_2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

so $\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 1 \end{bmatrix}$ is upper triangular.
 $\uparrow \quad \quad \quad \uparrow$
 $P_1 \quad \quad \quad P_2$

Multiplying by a perm. matrix on the right
permutes the columns.