

Math 5435 Homework #3

Section 1.6 # 5, 41

2.1 # 1, 10

1.6 #5 If $(A^2)^{-1} = B$, then $A^{-1} = AB$.

Hard part is to do this without assuming A^{-1} exists!
So we must write the proof without using A^{-1} .

PF We must show $(A)(AB) = I$ & $(AB)(A) = I$.

First one is easy: $A(AB) = A^2B = A^2(A^2)^{-1} = I$ *Done.*

Second one: $ABA = A(A^2)^{-1}A = A(A^2)^{-1}A \underbrace{A^2(A^2)^{-1}}_{\text{insert this since it equals } I} = A(A^2)^{-1}A^2A(A^2)^{-1}$
 $= A(A^2)^{-1}A^2A(A^2)^{-1}$ ($AA^2 = A^2A$ since they both equal A^3)
 $= A A(A^2)^{-1} = A^2(A^2)^{-1} = I$ *Done.*

1.6 #41

$$\begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

for which c is it not invertible?

Elimination:

$$\begin{bmatrix} 2 & c & c \\ 0 & c - \frac{1}{2}c^2 & c - \frac{1}{2}c^2 \\ 0 & 7 - 4c & -3c \end{bmatrix}$$

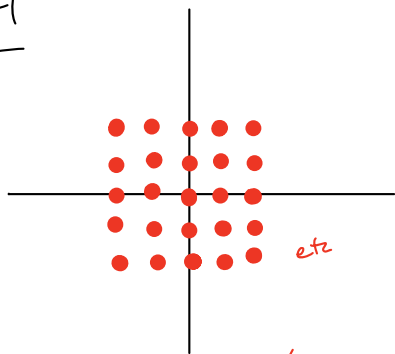
Not invertible when $c - c^2 = 0$,
since that makes this row trivial

Also not invertible if $7 - 4c = -3c$,
since then the bottom row will cancel
with the one above it.

$$\begin{array}{l} c - \frac{1}{2}c^2 = 0 \rightarrow \\ 7 - 4c = -3c \rightarrow \end{array} \boxed{\begin{array}{l} c=0, c=2 \\ c=7 \end{array}}$$

2.1 #1

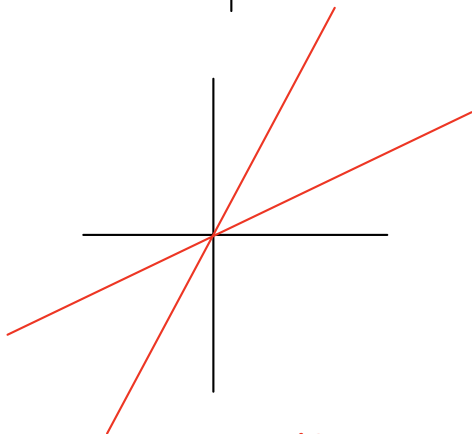
a)



$$\mathbb{Z}^2 = \{(x, y) \in \mathbb{R}^2 \mid x, y \in \mathbb{Z}\}$$

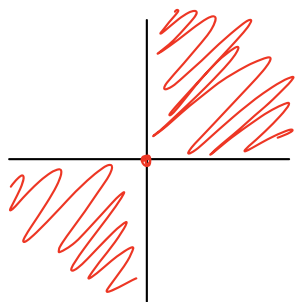
is closed under +,
but not scalar mult.

b)



Any union of lines thru $\vec{0}$

or



Two opposite quadrants
(including their boundary lines)

2.1 #10

$M_{2 \times 2}$

$\vec{0}$ is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}, \quad \frac{1}{2}A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \quad -A = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix}$$

The smallest subspace of A must include all
scalar mults of A , so it's:

$$\left\{ \begin{bmatrix} k & -k \\ k & -k \end{bmatrix} \mid k \in \mathbb{R} \right\}$$