

Math 5435 Homework #4

Section 2.1 #13

Section 2.2 #5a, 10, 51

2.1 #13

1. + associativity holds
2. + commutativity doesn't, since $f \circ g \neq g \circ f$ in general
3. $\text{id}_X(x) = x$ does satisfy the zero vector condition $f \circ \text{id} = f$
4. + inverses do not exist, since for some f there is no g with $f \circ g = \text{id}$.

5. $1f = f$ is true

6. $(c_1 c_2)f = c_1(c_2 f)$ is true

7. $c(f+g) = cf + cg$ is not true, since the 2 c 's on RHS are different from LHS.

8. $(c_1 + c_2)f = c_1 f + c_2 f$ is not true for same reason.

2.2 #5a

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 4 & 5 & 4 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array} \right] \quad \begin{array}{l} u = -3 - 2v \\ w = 2 \end{array}$$

↑
free

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -3-2v \\ v \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} + v \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

2.2 #10

2×3 system with sol $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + w \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

This comes from $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} w+1 \\ 3w+2 \\ w \end{bmatrix}$

which would come from $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -3 & 2 \end{array} \right]$

so $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

2.2 #51

$$N(A) = \left\{ c \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} \mid c \in \mathbb{R} \right\}$$

Solns are $\begin{bmatrix} 2c \\ 3c \\ c \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

↑
free var. c