Math 5435

HW #5

Section 2.3 #13, 20ab

Section 2.4 #5, 23

2.3 #13 $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$ $\quad U = \begin{bmatrix} \textcircled{1} & 1 & 0 \\ 0 & \textcircled{2} & 1 \\ 0 & 0 & 0 \end{bmatrix}$

pivot cols

All of $C(A), C(U), R(A), R(U)$ have dimension 2, since the rank is 2.

A & U have the same rowspace because row ops just recombine the rows, so will not change the rowspace.

2.3 #20ab in $\mathbb{R}^4$

a components all equal is $\left\{ c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, so it's dimension 1.

b components add to zero is

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ such that $x_1 + x_2 + x_3 + x_4 = 0$

a system with 3 free variables,

so this has dimension 3

2.4 #5

If $AB = 0$, then $C(B) \subset N(A)$

Pf Take $\vec{x} \in C(B)$, WTS $\vec{x} \in N(A)$, i.e. $A\vec{x} = \vec{0}$.

Since $\vec{x} \in C(B)$, $\vec{x} = B\vec{y}$ for some $\vec{y}$.

then $A\vec{x} = A(B\vec{y}) = (AB)\vec{y} = 0\vec{y} = \vec{0}$

Q.E.D.

2.4 #23

A is invertible 3×3.

then $R(A)$ & $C(A)$ are dimension 3, so all of $\mathbb{R}^3$

and $N(A)$ & $N(A^T)$ are just $\{\vec{0}\}$.

Bases $R(A)$ & $C(A)$: $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

$N(A)$ & $N(A^T)$: $\emptyset$

For $B = \begin{bmatrix} A & A \end{bmatrix}$,

the RREF is $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

$C(B)$ is 3 dim'l inside $\mathbb{R}^3$, so has basis $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

$R(B)$ is 3 dim'l inside $\mathbb{R}^6$, with basis $\left\{\begin{bmatrix}1\\0\\0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\\1\end{bmatrix}\right\}$

$N(B)$ is 3 dim'l inside $\mathbb{R}^6$, do free vars here

and will get basis $\left\{\begin{bmatrix}1\\0\\0\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\\-1\end{bmatrix}\right\}$

$N(B^T)$ is 0 dim'l inside $\mathbb{R}^3$, so has basis $\emptyset$.

(no free vars for B^T)