Math 5435 HW #7

Section 2.6 #8, 18

Section 3.1 #9, 44

2.6 #8 $T(f) = f(t) \cdot (2+3t)$

$T(1) = 2+3t$

$T(t) = 2t+3t^2$

$T(t^2) = 2t^2+3t^3$

$T(t^3) = 2t^3+3t^4$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

2.6 #18 $S \subset P_3 \qquad S = \{ p \mid \int_0^1 p(x)\,dx = 0 \}$

Closed under +: If $p, q \in S$, then $\int_0^1 p(x)\,dx = 0$ & $\int_0^1 q(x)\,dx = 0$

so $\int_0^1 (p+q)(x)\,dx = \int_0^1 p(x)\,dx + \int_0^1 q(x)\,dx = 0$

so $p+q \in S$.

closed under scalar mult.: If $p \in S$, then

$$\int_0^1 cp(x)\,dx = c\int_0^1 p(x)\,dx = c \cdot 0 = 0 \quad \checkmark$$

so $cp \in S$.

Basis: $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$$0 = \int_0^1 p(x)\,dx = a_0x + \frac{a_1}{2}x^2 + \frac{a_2}{3}x^3 + \frac{a_3}{4}x^4 \Big|_0^1$$

$$= a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4}$$

So $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$, do free vars:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{2} - \frac{a_2}{2} - \frac{a_3}{4} \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = a_1 \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} -1/3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} -1/4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{basis} = \left\{ -\tfrac{1}{2} + x,\ -\tfrac{1}{3} + x^2,\ -1/4 + x^3 \right\}$$

3.1 #9 $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ $R(A)$ is that plane

$R(A)^\perp = N(A)$ $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$= \left\{ z \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$ ← this line in $\mathbb{R}^3$.

3.1 #44 If $S \subset V$, show $S^\perp \supset V^\perp$

Pf Take $\vec{u} \in V^\perp$, will show $\vec{u} \in S^\perp$.

$\vec{u} \in V^\perp$ so $\vec{u} \perp \vec{v}$ for all $\vec{v} \in V$.

since $S \subset V$, this means $\vec{u} \perp \vec{v}$ for all $\vec{v} \in S$,

so $\vec{u} \in S^\perp$ as desired.

Shen.