Math 5435

Section 3.3 #3, 18, 23

Section 3.4 #4

3.3 #3 $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $A\vec{x} \approx \vec{b}$

$$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^T\vec{b} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^TA\hat{x} = A^T\vec{b}$$

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 2 & 1 \end{array}\right] \to \cdots \to \left[\begin{array}{cc|c} 1 & 0 & 1/3 \\ 0 & 1 & 1/3 \end{array}\right]$$

$$\hat{x} = \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$$

$$\vec{p} = A\hat{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix}$$

$$\vec{b} - \vec{p} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}$$

is $\perp$ to both cols of A.

3.3 #18 $y = C + Dt + Et^2$

a plug in points

$$3 = C + D + E$$
$$5 = C + 2D + E$$
$$6 = C + 3E$$
$$0 = C$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 0 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 5 \\ 3 & 5 & 3 \\ 5 & 3 & 11 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 26 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 & 5 \\ 3 & 5 & 4 \\ 5 & 4 & 11 \end{bmatrix} \hat{x} = \begin{bmatrix} 14 \\ 13 \\ 24 \end{bmatrix}$$

3.3 #23 Fit $y = C$ to points $y_1, \dots, y_m$.

Plug in pts: $y_1 = C$, $y_2 = C$, $\vdots$, $y_m = C$

$$A = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \qquad b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = [m]$$

$$A^T\vec{b} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} y_1 + \cdots + y_m \end{bmatrix}$$

$$A^T A \hat{x} = A^T \vec{b}$$

$$[m]\,\hat{x} = [y_1 + \cdots + y_m]$$

$$\hat{x} = \left[\frac{y_1 + \cdots + y_m}{m} \right]$$

Shown

3.4 #4 Q_1, Q_2 orth, then $Q_1 Q_2$ is orth:

$$(Q_1 Q_2)^T (Q_1 Q_2) = Q_2^T Q_1^T Q_1 Q_2$$

$$= Q_2^T \; I \; Q_2$$

$$= Q_2^T Q_2 = I$$

so $Q_1 Q_2$ is orth. Shown.

If $Q_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ & $Q_2 = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$,

then $Q_1 Q_2 = \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\sin\phi\cos\theta - \sin\theta\cos\phi \\ \cos\phi\sin\theta + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi \end{bmatrix}$

this is also rot by $(\theta + \phi)$, so

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

and $\sin(\theta + \phi) = \cos\phi\sin\theta + \cos\theta\sin\phi$

Cute!