# Math 1172 Catch-up Exam #1

## New ones (everybody do these)

**Question 13.** Please do the integral, and simplify any inverse trig functions in your answer:

$$\int \frac{x^3}{\sqrt{9-x^2}}\,dx$$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\sin\theta = \frac{x}{3}$

$$\int \frac{(3\sin\theta)^3}{\sqrt{9-9\sin^2\theta}}\, 3\cos\theta\, d\theta = \int \frac{27\sin^3\theta}{\sqrt{9\cos^2\theta}}\, 3\cos\theta\, d\theta$$

$$= \int \frac{27\sin^3\theta}{3\cos\theta}\, 3\cos\theta\, d\theta = 27\int \sin^3\theta\, d\theta$$

$$= 27\int \sin^2\theta \sin\theta\, d\theta = 27\int (1-\cos^2\theta)\sin\theta\, d\theta$$

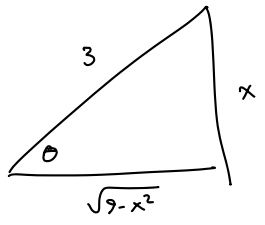
$u = \cos\theta$

$du = -\sin\theta$

$$= -27\int 1-u^2\, du = -27\left(u - \frac{1}{3}u^3\right) + C$$

$$= -27\left(\cos\theta - \frac{1}{3}(\cos\theta)^3\right) + C$$

$$= -27\left(\frac{\sqrt{9-x^2}}{3} - \frac{1}{3}\left(\frac{\sqrt{9-x^2}}{3}\right)^3\right) + C$$




$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

**Question 14.** Please do the integral:

$$\int \frac{x^3 - 3x^2 - 2x + 18}{x^2 - 2x - 3}$$

$$x^2 - 2x - 3 \;\overline{)\; x^3 - 3x^2 - 2x + 18}$$

$$x - 1$$

$$x^3 - 2x^2 - 3x$$

$$-x^2 + x + 18$$

$$-x^2 + 2x + 3$$

$$-x + 15$$

$$\int x - 1 + \frac{-x + 15}{x^2 - 2x - 3} \, dx$$

partial fraxx:

$$\frac{-x+15}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$= \frac{Ax + A + Bx - 3B}{}$$

$$-x + 15 = (A+B)x + A - 3B$$

$A + B = -1$

$A - 3B = 15$

$4B = -16$

$B = -4$

$A - 4 = -1$ $\quad A = 3$

$$\int x - 1 - \frac{1}{x-3} + \frac{3}{x+1} \, dx$$

$$= \frac{1}{2}x^2 - x - \ln|x-3| + 3\ln|x+1| + C$$

## Old ones (do on blank paper, only the ones you need)

**Question 1.** a) Please find $f'(\pi/2)$ and simplify all trig functions in the answer:

$$f(x) = x^2 \sin(3x)$$

b) Please do the integral:

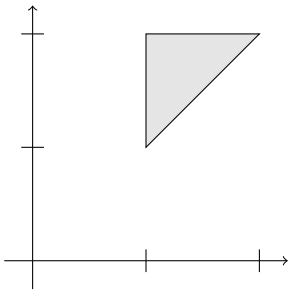
$$\int_1^3 \frac{5 - x^2}{x^4} dx$$

**Question 2.** Please do the integral:

$$\int_1^3 \frac{3x}{x^2 - 5} dx$$

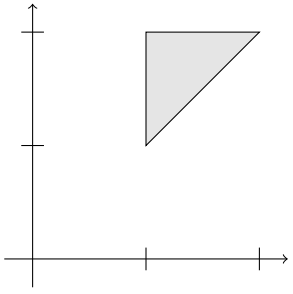
**Question 3.** Please find the area bounded between $y = -x$ and $y = x^2 - x - 1$.

**Question 4.**



Please use washers to find the volume obtained when revolving the shaded region around the $y$-axis. (Each mark along the axes is 1 unit.)

**Question 5.**



Please use shells to find the volume obtained when revolving the shaded region around the line $x = 3$. (Each mark along the axes is 1 unit.)

**Question 6.** a) Please find the derivative of $x \cos(\ln x)$

b) Please find the integral:

$$\int_1^3 \frac{x^2}{5x^3 - 1} dx$$

**Question 7.** a) Please find the derivative of:

$$f(x) = e^{\frac{\sin x}{x}}$$

b) Please find the integral:

$$\int x e^{1-x^2} dx$$

**Question 8.** a) Please find the derivative of $3^x + 3\log_4(x^2)$

b) Please find the integral:

$$\int x3^{x^2}\,dx$$

**Question 9.** a) Please find the derivative of $5x^2 + \tan^{-1}(2x)$

b) Please give the value of each expression, or say "impossible" if it's not possible to do by hand.

i) $\sin^{-1}\frac{1}{2}$

ii) $\tan^{-1} 2\pi$

iii) $\cos^{-1} 1$

iv) $\tan^{-1}\sqrt{3}$

**Question 10.** Please find the limit:

$$\lim_{x\to\infty} \frac{xe^x}{x^2 - 2x + 1}$$

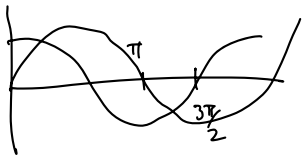
**Question 11.** Please find the integral:

$$\int \frac{x^2}{e^{2x}}\,dx$$

**Question 12.** Please find the integral

$$\int \cos^2 x \sin^3 x\,dx$$

**Question 1.** a) Please find $f'(\pi/2)$ and simplify all trig functions in the answer:




$$f(x) = x^2 \sin(3x)$$

$$f'(x) = x^2 \cdot \cos 3x \cdot 3 + \sin 3x \cdot 2x$$

$$f'(\pi/2) = (\pi/2)^2 \cos\tfrac{3\pi}{2} \cdot 3 + \sin\tfrac{3\pi}{2} \cdot 2 \cdot \pi/2$$

$$= (\tfrac{\pi}{2})^2 \cdot 0 \cdot 3 + -1 \cdot \pi = -\pi$$

b) Please do the integral:

$$\int_1^3 \frac{5 - x^2}{x^4} dx = \int (5 - x^2) x^{-4} \, dx$$

$$= \int_1^3 5x^{-4} - x^{-2} \, dx = \frac{5}{-3} x^{-3} - -x^{-1} \Big|_1^3$$

$$= -\frac{5}{3} \cdot 3^{-3} + 3^{-1} - \left( -\frac{5}{3} \cdot 1^{-3} + 1^{-1} \right)$$

**Question 2.** Please do the integral:

$$\int_1^3 \frac{3x}{(x^2+5)^7} \, dx$$

$u = x^2 + 5$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$$= 3 \int \frac{x}{u^7} \cdot \frac{1}{2} du = \frac{3}{2} \int \frac{1}{u^7} x\,dx$$

$$= \frac{3}{2} \int u^{-7} du = \frac{3}{2} \cdot \frac{1}{-6} u^{-6}$$

$$= -\frac{3}{4} (x^2+5)^{-6} \Big|_1^3 = -\frac{3}{4} \left( 3^2 + 5 \right)^{-6} - \frac{-3}{4} \left( 1^2 + 5 \right)^{-6}$$

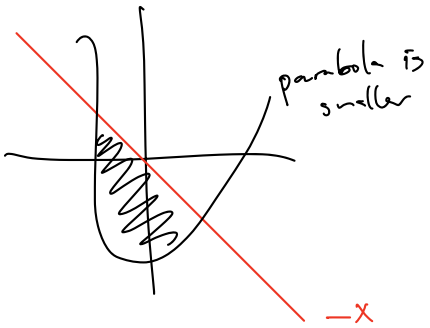
**Question 3.** Please find the area bounded between $y = -x$ and $y = x^2 - x - 1$.

intersections: $-x = x^2 - x - 1$

$0 = x^2 - 1$

$0 = (x+1)(x-1)$

$x = 1, -1$

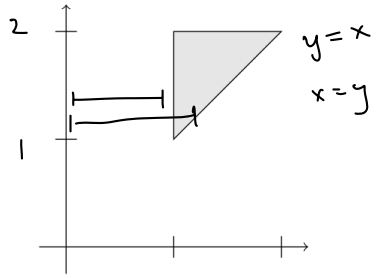



$$\int_{-1}^{1} -x - (x^2 - x - 1)\,dx = \int_{-1}^{1} -x - x^2 + x + 1\,dx$$

$$= \int_{-1}^{1} 1 - x^2\,dx = x - \frac{1}{3}x^3 \Big|_{-1}^{1}$$

$$= 1 - \frac{1}{3} - \left(-1 - \frac{1}{3}(-1)^3\right)$$

**Question 4.**




inner = 1

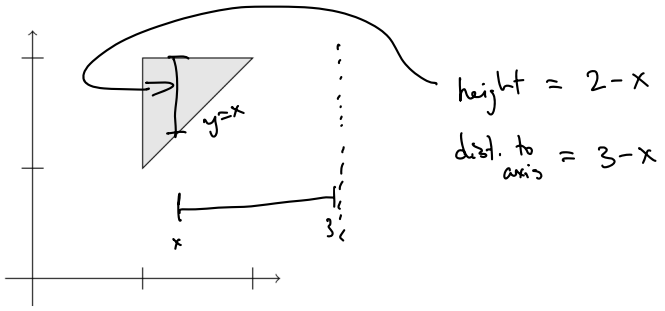
outer = y − 1

Please use washers to find the volume obtained when revolving the shaded region around the $y$-axis. (Each mark along the axes is 1 unit.)

$$\pi \int_1^2 (y-1)^2 - 1^2\,dy = \pi \int_1^2 y^2 - 2y + 1 - 1\,dy$$

$$= \pi \int_1^2 y^2 - 2y\,dy = \pi\left(\frac{1}{3}y^3 - y^2\right)\Big|_1^2 = \pi\left(\frac{1}{3}\cdot 2^3 - 2^2\right) - \pi\left(\frac{1}{3}\cdot 1^3 - 1^2\right)$$

**Question 5.**




Please use shells to find the volume obtained when revolving the shaded region around the line $x = 3$. (Each mark along the axes is 1 unit.)

$$2\pi \int_1^2 (3-x)(2-x)\,dx$$

$$= 2\pi \int_1^2 6 - 5x + x^2\,dx = 2\pi \left(6x - \frac{5}{2}x^2 + \frac{1}{3}x^3\right)\Big|_1^2$$

$$= 2\pi \left(6\cdot 2 - \frac{5}{2}\cdot 2^2 + \frac{1}{3}\cdot 2^3\right) - 2\pi \left(6 - \frac{5}{2} + \frac{1}{3}\right)$$

**Question 6.** a) Please find the derivative of $x\cos(\ln x)$

$$f'(x) = x \cdot -\sin(\ln x)\cdot \frac{1}{x} + \cos(\ln x)$$

b) Please find the integral:

$$\int_1^3 \frac{x^2}{5x^3 - 1}\,dx$$

$u = 5x^3 - 1$

$du = 15x^2\,dx$

$\frac{1}{15}du = x^2\,dx$

$$= \int \frac{1}{u}\cdot\frac{1}{15}\,du = \frac{1}{15}\int \frac{1}{u}\,du = \frac{1}{15}\ln|u|$$

$$= \frac{1}{15}\ln|5x^3 - 1|\Big|_1^3 = \frac{1}{15}\ln|5\cdot 3^3 - 1| - \frac{1}{15}\ln|5\cdot 1^3 - 1|$$

**Question 7.** a) Please find the derivative of:

$$f(x) = e^{\frac{\sin x}{x}}$$

$$f'(x) = e^{\frac{\sin x}{x}} \cdot \frac{x \cdot \cos x - \sin x}{x^2}$$

b) Please find the integral:

$$\int x e^{1-x^2}\, dx$$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= -\frac{1}{2}\int e^u\, du = -\frac{1}{2}e^u + C$$

$$= -\frac{1}{2}e^{1-x^2} + C$$

**Question 8.** a) Please find the derivative of $3^x + 3\log_4(x^2)$

$$3^x \ln 3 + 3 \cdot \frac{1}{x^2 \ln 4} \cdot 2x$$

b) Please find the integral:

$$\int x 3^{x^2}\, dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int 3^u\, du = \frac{1}{2} \cdot \frac{1}{\ln 3} 3^u + C$$

$$= \frac{1}{2}\frac{1}{\ln 3} 3^{x^2} + C$$

**Question 9.** a) Please find the derivative of $5x^2 + \tan^{-1}(2x)$

$$10x + \frac{1}{(2x)^2+1} \cdot 2$$

b) Please give the value of each expression, or say "impossible" if it's not possible to do by hand.

i) $\sin^{-1}\frac{1}{2}$ $= \pi/6$

ii) $\tan^{-1} 2\pi$ impossible

iii) $\cos^{-1} 1$ $= 0$

iv) $\tan^{-1}\sqrt{3}$ $= \pi/6$

**Question 10.** Please find the limit:

$$\lim_{x\to\infty} \frac{xe^x}{x^2 - 2x + 1} \qquad \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{xe^x + e^x}{2x - 2} \qquad \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{xe^x + e^x + e^x}{2} \qquad \frac{\infty}{2}$$

limit does not exist

**Question 11.** Please find the integral:

$$\int \frac{x^2}{e^{2x}}\,dx$$

$$\int x^2 e^{-2x}\,dx \qquad u = x^2 \quad du = 2x\,dx$$

$$dv = e^{-2x}dx \quad v = -\frac{1}{2}e^{-2x}$$

$$= uv - \int v\,du = x^2 \cdot \frac{1}{-2}e^{-2x} - \int \frac{1}{-2}e^{-2x} \cdot 2x\,dx$$

$$= -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\,dx \qquad u = x \quad du = dx$$

$$dv = e^{-2x} \quad v = \frac{-1}{2}e^{-2x}$$

$$= -\frac{1}{2}x^2e^{-2x} + x \cdot \frac{1}{-2}e^{-2x} - \int \frac{-1}{2}e^{-2x} \cdot dx$$

$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} + \frac{1}{2} \cdot \frac{1}{-2}e^{-2x} + C$$

**Question 12.** Please find the integral

$$\int \cos^2 x \sin^3 x \, dx$$

$$\int \cos^2 x \sin^2 x \sin x \, dx \qquad u = \cos x, \quad du = -\sin x \, dx$$

$$= \int \cos^2 x (1 - \cos^2 x) \sin x \, dx$$

$$= -\int u^2 (1 - u^2) \, du$$

$$= -\int u^2 - u^4 \, du = -\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right) + C$$

$$= -\left(\frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right) + C$$