Math 1172 Homework #5

Section 6.8 #50

7.1 #15, 57

7.2 #4

6.8 #50

$$\lim_{x\to \pi/2^-} \cos x \sec 5x = \lim_{x\to \pi/2^-} \frac{\cos x}{\cos 5x} = \frac{\cos \pi/2}{\cos 5\pi/2} = \frac{0}{0}$$

$$\overset{\#}{=} \lim_{x\to \pi/2} \frac{-\sin x}{-\sin 5x \cdot 5} = \frac{-\sin \pi/2}{-\sin 5\pi/2 \cdot 5} = \frac{-1}{-1 \cdot 5} = \frac{1}{5}$$

7.1 #15

$$\int t^4 \ln t \, dt \qquad u = \ln t \qquad dv = t^4 \, dt$$

$$du = \frac{1}{t} dt \qquad v = \frac{1}{5} t^5$$

$$= uv - \int v \, du$$

$$= \ln t \cdot \frac{1}{5} t^5 - \int \frac{1}{5} t^5 \cdot \frac{1}{t} dt$$

$$= \frac{1}{5} t^5 \ln t - \frac{1}{5} \int t^4 dt$$

$$= \frac{1}{5} t^5 \ln t - \frac{1}{5} \cdot \frac{1}{5} t^5 + C$$

7.1 #57

$$\int (\ln x)^n dx \qquad u = (\ln x)^n \qquad dv = dx$$

$$du = n(\ln x)^{n-1} \cdot \frac{1}{x} dx \qquad v = x$$

$$= uv - \int v \, du = (\ln x)^n \cdot x - \int x \cdot n (\ln x)^{n-1} \cdot \frac{1}{x} dx$$

$$= x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

7.2 # 15

$$\int_0^{\pi/4} \sin^5 x\,dx = \int_0^{\pi/4} \sin^4 x \sin x\,dx = \int_0^{\pi/2} (\sin^2 x)^2 \sin x\,dx$$

$$= \int_0^{\pi/4} (1-\cos^2 x)^2 \sin x\,dx \qquad u = \cos x,\ -du = \sin x\,dx$$

$$= -\int_0^{\pi/4} (1-u^2)^2\,du$$

$$= -\int_0^{\pi/4} 1 - 2u^2 + u^4\,du$$

$$= -\left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right)$$

$$= -\left(\cos x - \frac{2}{3}(\cos x)^3 + \frac{1}{5}(\cos x)^5\right)\Bigg|_0^{\pi/4}$$

$$= -\left(\cos\pi/4 - \frac{2}{3}(\cos\pi/4)^3 + \frac{1}{5}(\cos\pi/4)^5\right) - \left(-\left(\cos 0 - \frac{2}{3}(\cos 0)^3 + \frac{1}{5}(\cos 0)^5\right)\right)$$

$$= -\left(\frac{1}{\sqrt{2}} - \frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{2}}\right)^5\right) + \left(1 - \frac{2}{3} + \frac{1}{5}\right)$$