# Math 1172 HW #6

7.3 #9, 10, 12, 14

**#9** $\int x^3 \sqrt{16+x^2}\, dx$ $\quad x = 4\tan\theta$, $dx = 4\sec^2\theta\, d\theta$

$= \int 4^3 \tan^3\theta \sqrt{16+16\tan^2\theta}\; 4\sec^2\theta\, d\theta$

$= 4^4 \int \tan^3\theta \sec^2\theta \sqrt{16\sec^2\theta}\, d\theta$

$= 4^4 \int \tan^3\theta \sec^2\theta \cdot 4\sec\theta\, d\theta$

$= 4^5 \int \tan^3\theta \sec^3\theta\, d\theta = 4^4 \int \frac{\sin^3\theta}{\cos^6\theta}\, d\theta$

$= 4^5 \int \frac{\sin^2\theta}{\cos^6\theta} \sin\theta\, d\theta$

$= 4^5 \int \frac{1-\cos^2\theta}{\cos^6\theta} \cdot \sin\theta\, d\theta$ $\quad u = \cos\theta$, $du = -\sin\theta\, d\theta$

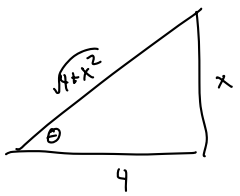
$= 4^5 \int \frac{1-u^2}{u^6} \cdot -du = -4^4 \int (1-u^2)u^{-6}\, du$

$= -4^5 \int u^{-6} - u^{-4}\, du = -4^4 \left(\frac{1}{-5}u^{-5} - \frac{1}{-3}u^{-3}\right) + C$

$= -4^5 \left(\frac{1}{-5}(\cos\theta)^{-5} + \frac{1}{3}(\cos\theta)^{-3}\right) + C$

$= -4^5 \left(\frac{1}{-5}\left(\frac{4}{\sqrt{4+x^2}}\right)^{-5} + \frac{1}{3}\left(\frac{4}{\sqrt{4+x^2}}\right)^{-3}\right) + C$

$x/4 = \tan\theta$




$\cos\theta = \frac{4}{\sqrt{4+x^2}}$

#10 $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$ $\quad x = 3\sin\theta$ $\quad dx = 3\cos\theta\,d\theta$ $\quad \theta = \sin^{-1}\frac{x}{3}$

$$= \int \frac{(3\sin\theta)^2}{\sqrt{9-(3\sin\theta)^2}} \cdot 3\cos\theta\,d\theta$$

$$= \int \frac{9\sin^2\theta}{\sqrt{9\cos^2\theta}}\,3\cos\theta\,d\theta = \int \frac{9\sin^2\theta}{3\cos\theta}\,3\cos\theta\,d\theta$$

$$= 9\int \sin^2\theta\,d\theta = 9\int \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{9}{2}\int 1-\cos 2\theta\,d\theta = \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{1}{2}\sin\left(2\sin^{-1}\frac{x}{3}\right)\right) + C$$

#12 $\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$ $\quad x = 6\sin\theta$ $\quad dx = 6\cos\theta\,d\theta$

$x=0$: $0 = 6\sin\theta$, $\sin\theta = 0$, $\theta = 0$

$x=3$: $3 = 6\sin\theta$, $\frac{1}{2} = \sin\theta$, $\theta = \pi/6$

$$= \int_0^{\pi/6} \frac{6\sin\theta}{\sqrt{36-36\sin^2\theta}}\,6\cos\theta\,d\theta$$

$$= \int_0^{\pi/6} \frac{6\sin\theta}{\sqrt{36\cos^2\theta}}\,6\cos\theta\,d\theta = \int_0^{\pi/6} \frac{6\sin\theta}{6\cos\theta}\,6\cos\theta\,d\theta$$

$$= \int_0^{\pi/6} 6\sin\theta\,d\theta = -6\cos\theta\Big|_0^{\pi/6} = -6\cos\pi/6 - (-6\cos 0)$$

$$= -6\cdot\frac{\sqrt{3}}{2} + 6$$

#14 $\int \frac{1}{t^2\sqrt{t^2-16}}\,dt$ $\quad t = 4\sec\theta$ $\quad dt = 4\sec\theta\tan\theta\,d\theta$ $\quad \sec\theta = \frac{t}{4}$

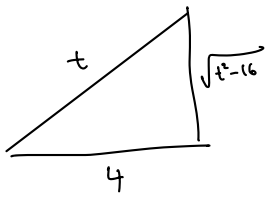
$$= \int \frac{1}{16\sec^2\theta\sqrt{16\sec^2\theta - 16}} \cdot 4\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{16\sec^2\theta \cdot 4\sqrt{\tan^2\theta}} \cdot 4\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{16\sec^2\theta \cdot 4\tan\theta} \cancel{4\sec\theta\tan\theta}\,d\theta$$

$$= \frac{1}{16}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{16}\int \cos\theta\,d\theta = \frac{1}{16}\sin\theta + C$$

$\sec\theta = t/4$




$$= \frac{1}{16}\,\frac{\sqrt{t^2-16}}{t} + C$$

$\sin\theta = \frac{\sqrt{t^2-16}}{t}$