Math 1172 HW #7

#9, 12, 17, 18

#9

$$\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$$

$$5x+1 = A(x-1) + B(2x+1)$$

$$5x+1 = (A+2B)x - A + B$$

$$\begin{aligned} A+2B &= 5 \\ -A+B &= 1 \end{aligned} \Big\} \rightarrow \quad 3B = 6, \quad B = 2 \qquad -A+2 = 1, \quad A = 1$$

$$\int \frac{5x+1}{(2x+1)(x-1)}\,dx = \int \frac{1}{2x+1} + \frac{2}{x-1}\,dx$$

$$= \int \frac{1}{2}\,\frac{1}{x+1/2} + 2\,\frac{1}{x-1}\,dx$$

$$= \frac{1}{2}\ln|x+1/2| + 2\ln|x-1| + C$$

#12

$$\frac{x-4}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$x-4 = Ax-2A+Bx-3B$$

$$x-4 = (A+B)x - 2A - 3B$$

$$\begin{aligned} A+B &= 1 \\ -2A-3B &= -4 \end{aligned} \Big\} \quad -B = -2, \quad B = 2 \qquad A+2 = 1, \quad A = -1$$

$$\int_0^1 \frac{x-4}{x^2-5x+6}\,dx = \int_0^1 \frac{-1}{x-3} + \frac{2}{x-2}\,dx = -\ln|x-3| + 2\ln|x-2| \Big|_0^1$$

$$= -\ln|1-3| + 2\ln|1-2| - \left(-\ln|0-3| + 2\ln|0-2|\right)$$

$$= -\ln 2 + 2\ln 1 + \ln 3 - 2\ln 2 = -3\ln 2 + \ln 3$$

#17

$$\frac{4y^2-7y-12}{y(y+2)(y-3)} = \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3}$$

$$4y^2-7y-12 = A(y+2)(y-3) + B\cdot y(y-3) + C\cdot y(y+2)$$

$$= A(y^2-y-6) + B(y^2-3y) + C(y^2+2y)$$

$$4y^2-7y-12 = (A+B+C)y^2 + (-A-3B+2C)y + -6A$$

$$\left.\begin{array}{l} A+B+C=4 \\ -A-3B+2C=-7 \end{array}\right\} \longrightarrow \begin{array}{l} 2+B+C=4 \\ -2-3B+2C=-7 \end{array}$$

$$-6A=-12 \rightarrow A=2$$

$$B+C=2$$
$$-3B+2C=-5$$

$$5C=1$$
$$C=1/5$$

$$B=9/5$$

$$\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy = \int_1^2 \frac{2}{y} + \frac{9/5}{y+2} + \frac{1/5}{y-3}\,dy$$

$$= 2\ln|y| + \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3| \Big|_1^2$$

$$= 2\ln 2 + \frac{9}{5}\ln 4 + \frac{1}{5}\ln 1 - \left(2\ln 1 + \frac{9}{5}\ln 3 + \frac{1}{5}\ln 2\right)$$

#18

$$\frac{3x^2+6x+2}{x^2+3x+2} = 3 + \frac{-3x-4}{x^2+3x+2}$$

$$\begin{array}{r} 3 \\ x^2+3x+2 \overline{)\,3x^2+6x+2} \\ \underline{3x^2+9x+6} \\ -3x-4 \end{array}$$

$$\frac{-3x-4}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$-3x-4 = Ax+2A+Bx+B$$

$$= (A+B)x + 2A+B$$

$$-3 = A+B$$

$$-4 = 2A+B$$

$$1 = -A \qquad -3 = -1+B$$

$$A=-1 \qquad B=-2$$

$$\int_1^2 \frac{3x^2+6x+2}{x^2+3x+2} = \int_1^2 3 + \frac{-1}{x+1} + \frac{-2}{x+2}\,dx$$

$$= 3x - \ln|x+1| - 2\ln|x+2| \Big|_1^2$$

$$= 6 - \ln 3 - 2\ln 4 - \left(3 - \ln 2 - 2\ln 3\right)$$