Math 1172 HW #8

Section 7.8 # 22, 39

Section 11.1 # 20, 80

7.8 # 22

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx \qquad u = -1/x = -x^{-1}$$
$$du = x^{-2}\,dx$$

$$= \lim_{t\to\infty} \int_1^t e^u\,du = \lim_{t\to\infty} e^u = \lim_{t\to\infty} e^{-1/x}\Big|_1^t$$

$$= \lim_{t\to\infty} e^{-1/t} - e^1 = e^{-0} - e^1 = \boxed{1-e}$$

7.8 # 39

$$\int_{-2}^3 \frac{1}{x^4}\,dx = \int_{-2}^0 x^{-4}\,dx + \int_0^3 x^{-4}\,dx$$

$$\lim_{t\to 0} \int_{-2}^t x^{-4}\,dx = \lim_{t\to 0} -\frac{1}{3}x^{-3}\Big|_{-2}^t = \lim_{t\to 0} \frac{-1}{3}t^{-3} - \frac{-1}{3}(-2)^{-3}$$

$$= \lim_{t\to 0} \frac{-1}{3}\cdot\frac{1}{t^3} + \frac{1}{3}(-2)^{-3}$$

$\downarrow$

DNE

The integral diverges.

 # 20 $5, 8, 11, 14, \ldots$

Start with 5, add 3 more each time

$$a_n = 5 + 3(n-1)$$

or $2 + 3n$

\# 80 $a_n = \frac{1-n}{2+n}$ is decreasing

WTS $a_{n+1} < a_n$

which is the same as:

WTS $\frac{1-(n+1)}{2+(n+1)} < \frac{1-n}{2+n}$

WTS $\frac{1-n-1}{2+n+1} < \frac{1-n}{2+n}$

WTS $\frac{-n}{n+3} < \frac{1-n}{2+n}$

WTS $-n(2+n) < (1-n)(n+3)$

$-\cancel{2n} - \cancel{n^2} < -\cancel{n^2} - \cancel{2n} + 3$

$0 < 3$

and this is obvious.

So it's decreasing