Math 1172 HW #10

13, 17, 19, 23

#13 $1 + \frac{1}{8} + \frac{1}{27} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^3}$

$$\int_1^\infty \frac{1}{x^3}\,dx = \lim_{t\to\infty} \int_1^t x^{-3}\,dx = \lim_{t\to\infty} -\frac{1}{2} x^{-2} \Big|_1^t$$

$$= \lim_{t\to\infty} \frac{-1}{2} t^{-2} - \frac{-1}{2}\cdot 1^{-2} = 0 + \frac{1}{2}$$

the integral converges, so the series converges

#17 $\sum \frac{\sqrt{n}+4}{n^2}$

$$\int_1^\infty \frac{x^{1/2}+4}{x^2}\,dx = \lim_{t\to\infty} \int_1^t x^{-3/2} + 4x^{-2}\,dx$$

$$= \lim_{t\to\infty} -2x^{-1/2} - 4x^{-1} \Big|_1^t$$

$$= \lim_{t\to\infty} -\underset{\downarrow\atop 0}{2t^{-1/2}} - \underset{\downarrow\atop 0}{4t^{-1}} - (-2-4) = 6$$

the integral converges, so the series converges

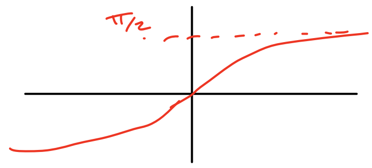
#19 $\sum \frac{1}{n^2+4}$

$$\int_1^\infty \frac{1}{x^2+4}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{4}\,\frac{1}{\frac{x^2}{4}+1}\,dx = \lim_{t\to\infty} \frac{1}{4}\int_1^t \frac{1}{(\frac{x}{2})^2+1}\,dx$$

$u = x/2$
$du = \frac{1}{2}dx$

$$= \lim_{t\to\infty} \frac{1}{4}\cdot 2 \arctan u = \lim_{t\to\infty} \frac{1}{2}\arctan\frac{x}{2}\Big|_1^t$$

$$= \lim_{t\to\infty} \frac{1}{2}\arctan\frac{t}{2} - \frac{1}{2}\arctan \tfrac{1}{2}$$



$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\arctan \tfrac{1}{2}$$

the integral converges, so <u>the series converges</u>

<u>#23</u> $\sum \frac{1}{n \ln n}$

$$\int_1^\infty \frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x\ln x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \lim_{t\to\infty}\int \frac{1}{u}\,du = \lim_{t\to\infty} \ln|u|$$

$$= \lim_{t\to\infty} \ln|\ln x|\Big|_1^t = \lim_{t\to\infty} \underset{\substack{\downarrow\\ \infty}}{\ln|\ln t|} - \ln|\ln 1|$$

integral diverges, so the series <u>diverges</u>.