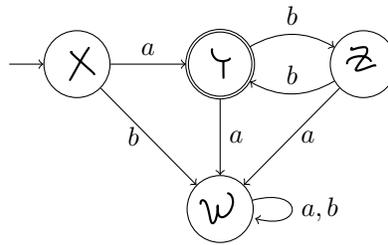


Math 3342 Exam #1

Question 1. For the following DFA, please find a formula for all strings which are accepted, and then prove that they are accepted.



ab^{2n}

Thm $\delta^*(X, ab^{2n}) = Y$

pf By induction on n .

Base case $n=0$ WTS $\delta^*(X, a) = Y$, clear on the diagram.

Induction Assume $\delta^*(X, ab^{2k}) = Y$, WTS $\delta^*(X, ab^{2(k+1)}) = Y$.

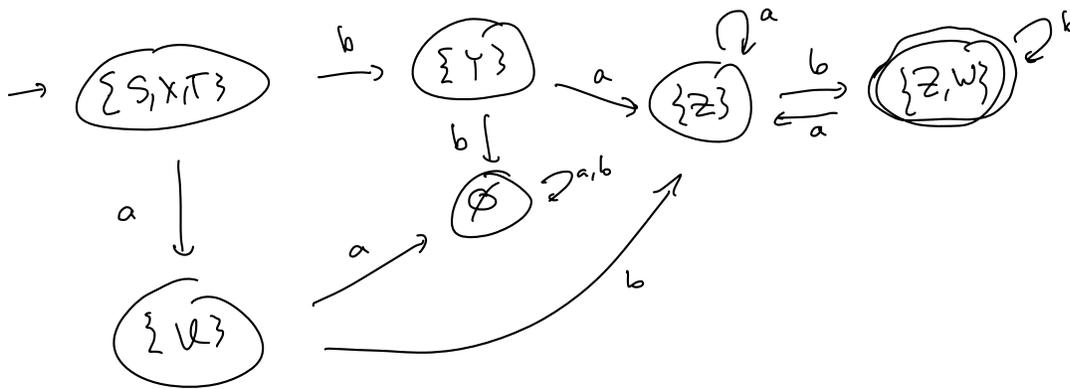
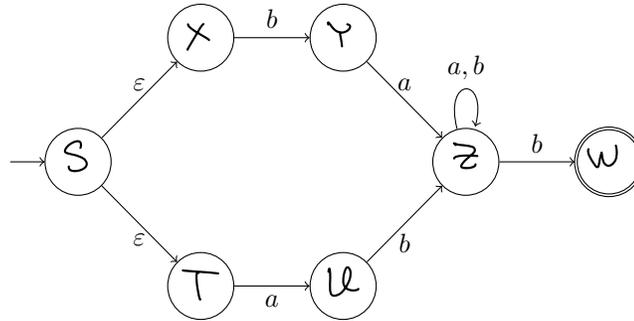
We have:

$$\delta^*(X, ab^{2(k+1)}) = \delta^*(X, ab^{2k+2}) = \delta^*(X, ab^{2k}b^2)$$

$$= \delta^*(\delta^*(X, ab^{2k}), b^2)$$

$$\stackrel{IH}{=} \delta^*(Y, b^2) = Y \quad \text{QED!}$$

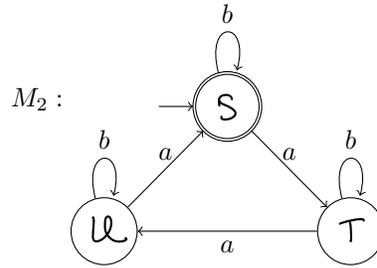
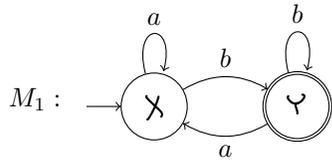
Question 2. a) Please convert this NFA to a DFA using the subset construction:



b) The subset construction is a method for converting an NFA to a DFA. But we never discussed a method to convert a DFA to an NFA. Why not? Explain in a sentence or two. (You don't need to write a lot.)

A DFA diagram is already an NFA,
 so we don't need to convert anything.

Question 3. This whole page is about these two DFAs M_1 and M_2 :

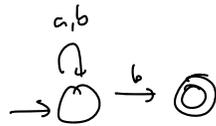


a) Please describe the sets $L(M_1)$ and $L(M_2)$ using set theory notation.

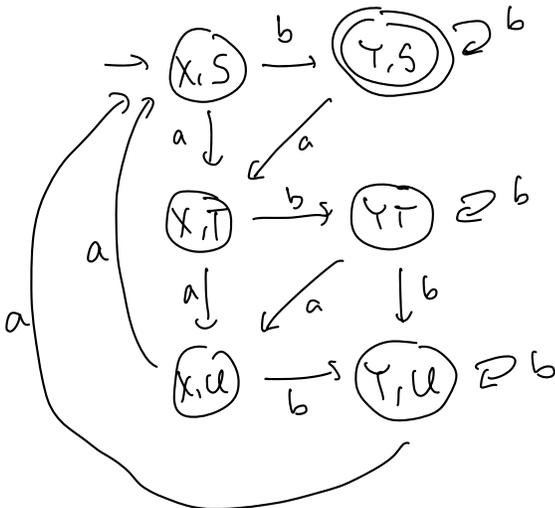
$$L(M_1) = \{ x b \mid x \in \{a, b\}^* \}$$

$L(M_2)$ is the set of all strings where # of a's is div. by 3.

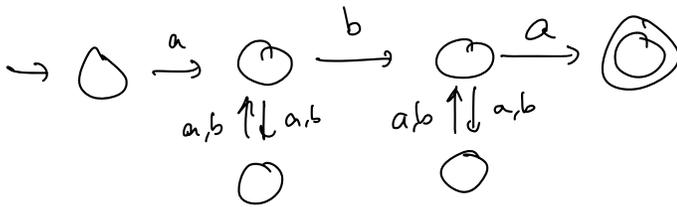
b) Please draw an NFA which has the same language as M_1 but uses fewer arrows.



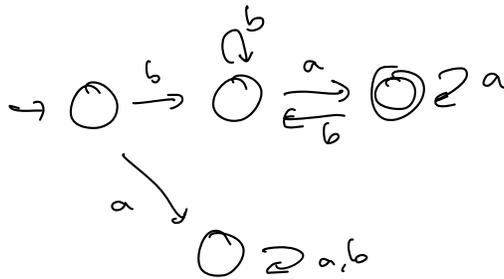
c) Please use the subset construction to make a DFA for $L(M_1) \cap L(M_2)$.



Question 4. a) Please make an NFA for the language $\{axbya \mid x, y \in \{a, b\}^* \text{ both have even length}\}$



b) Please make a DFA for the language $\{x^2 \mid x \in \{a, b\}^*\}$



c) Please make a DFA or NFA (your choice) for the language $\{a^n x b^m \mid m > 1, x \in \{a, b\}^*\}$. Say specifically whether your machine is a DFA or NFA.

NFA:

