Math 3342 Exam #2

Question 1. (7 points each) For each language, either give a regular expression or a context-free grammar (your choice)

a) $\{ab^n(ab)^k\}$

$ab^*(ab)^*$

b) $\{a^nb^ma^mb^n\}$

$S \to aSb \mid T$
$T \to bTa \mid \varepsilon$

c) $\{a^nxb^n \mid x \in \{a,b\}^* \text{ and } |x| \text{ is divisible by } 3\}$

$S \to aSb \mid T$
$T \to XXXT \mid \varepsilon$
$X \to a \mid b$

d) The language of bracketed lists of binary strings, like:

$$[010, 101, 1, 00, 1]$$

There should be: brackets surrounding finitely many (zero or more) binary (nonempty) strings, with a comma in between each of them.
(Make sure you do not allow stray commas on the ends, or two commas with nothing in between.)

$[\,] + [(0+1)(0+1)^*(\,,(0+1)(0+1)^*)^*]$

$S \to [T]$
$T \to B \mid B,T$
$B \to 0B \mid 1B \mid 0 \mid 1$

Question 2. Let $L = \{a^n b a^{2n}\}$

a) (10 points) Please make a stack machine "by hand" (don't convert from grammar) which accepts L, and give a derivation for $abaa$.

read	pop	push
a	S	SXX
b	S	ε
a	X	ε

$(abaa, S) \mapsto (baa, SXX) \mapsto (aa, XX) \mapsto (a, X) \mapsto (\varepsilon, \varepsilon)$ ✓

b) (6 points) Please make a CFG for L, and show a grammar derivation for a^2ba^4.

$S \to aSaa \mid b$

$S \Rightarrow aSaa \Rightarrow aaSaaaa \Rightarrow aabaaaa$

Question 3. (10 points) Please show that $L = \{(ab)^n a^n b^n\}$ is not regular.

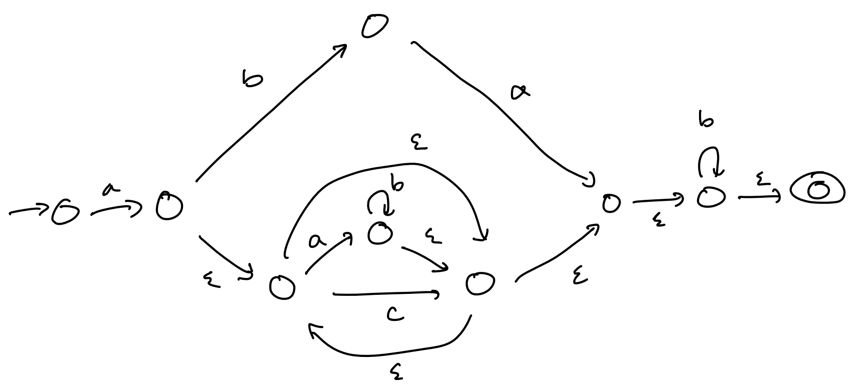
Let $D_i = \frac{d}{d(ab)^i} L$

$= \{ (ab)^{n-i} a^n b^n \}$

These are all different for various i,

so L is not regular.

Question 4. (10 points) Please make an NFA equivalent to this regular expression: $a((ab^* + c)^* + ba)b^*$



Question 5. Here is a bogus proof that the language $L = \{a^n b^m\}$ is nonregular:

Let $D_i = \frac{d}{da^i} L = \{a^{n-i} b^m\}$. *Then these sets are all different for various* i, *so* L *has infinitely many derivatives, so* L *is not regular. Shewn!*

a) (6 points) Please show that L is in fact regular.

L has a regular expression

a^*b^*

so it's regular.

b) (10 points) Please identify the specific error in the bogus proof above.

Those sets are not all different!

Since n & m & i can all be anything,

these sets are all the same as $\{a^n b^m\}$.

Question 6. (5 points each) Here is a stack machine:

read	pop	push
ε	S	XY
a	X	XT
a	Y	YR
ε	X	ε
ε	Y	ε
b	T	ε
b	R	ε

a) Please give a stack machine derivation showing that *abab* is accepted.

$(abab, S) \mapsto (abab, XY) \mapsto (bab, XTY) \mapsto (bab, TY)$
$\mapsto (ab, Y) \mapsto (b, YR) \mapsto (b, R) \mapsto (\varepsilon, \varepsilon)$ ✓

b) Please show that *aab* is not accepted.

$(aab, S) \mapsto (aab, XY) \mapsto (ab, XTY) \mapsto (b, XTTY)$
$\mapsto (b, TTY) \mapsto (\varepsilon, TY)$ stuck!

could've also done ε X ε earlier, but this would also get stuck.

c) What is the language accepted by the stack machine?

$a^n b^n a^m b^m$

d) Please give a grammar equivalent to this stack machine.

$S \to XY$
$X \to aXT \mid \varepsilon$
$Y \to aYR \mid \varepsilon$
$T \to b \quad R \to b$

simplified: $S \to XX$
$X \to aXb \mid \varepsilon$