# Math 3342 Exam #2 practice

(This is longer than our actual test will be– I'm just trying to cover all the topics here)

**Question 1.** For each language, give a regular expression:

a) $\{a^n b^k\}$

b) The set of all strings on $\Sigma = \{a, b\}$ in which every $a$ is followed immediately by $b$.

c) The set of all strings on $\Sigma = \{a, b, c\}$ where no $c$ appears after any $a$. (So $cbab$ is in the language, but $babc$ is not in the language because it has a $c$ after an $a$.)

d) The set of all strings on $\Sigma = \{a, b, c\}$ where every $a$ appears with $c$ on both sides immediately before and after it.

**Question 2.** Please convert this regular expression to NFA: $((aa + bab)^* + bc^*)^* b$.

**Question 3.** For each language, please make a grammar:

a) $\{(ab)^n a^n\}$

b) $\{a^n b^k \mid k = n \text{ or } k = n + 2\}$

c) $\{a^n b^k c^m\}$

d) $\{a^{2n} b a^k b^n\}$

e) Which of your grammars above are context-free, and which are not? Briefly explain (enough so that I know that you know what "context free" means).

f) One of those languages above is regular. Decide which one, and make a regular expression for it.

**Question 4.** a) Please make a grammar for the language of strings that look like:

`[aaa][aaaa][aa][][a]`

These are strings consisting of zero or more blocks, each of which has brackets around 0 or more a's. (The empty string is part of this language.)

b) Please make a regular expression for the same language.

**Question 5.** Please give an example of a nonempty set $L$ such that some derivative of $L$ equals $L$. Write clearly which derivative of $L$ you are using.

**Question 6.** Please show that $L = \{xax^2 \mid x \in \{a, b\}^*\}$ is not a regular language.

**Question 7.** Here is a stack machine:

| | read | pop | push |
|---|---|---|---|
| 1. | $a$ | $S$ | $SXX$ |
| 2. | $b$ | $X$ | $\varepsilon$ |
| 3. | $\varepsilon$ | $S$ | $\varepsilon$ |

a) Please show that $aabbbb$ is accepted by this stack machine.

b) Please show that $ab$ is not accepted.

c) Please convert the stack machine to an equivalent grammar.

d) Please describe the language of this machine in words or using set theory notation.

e) If we change the "push $\varepsilon$" in rule 3 to "push $X$", then what will the language of this new machine be?

f) If we change the "push $\varepsilon$" in rule 2 to "push $X$" (but keep rule 3 like it originally was), then what will the language of this new machine be?

**Question 8.** Please show that $L = \{a^k b^n a^n\}$ is not a regular language.

**Question 9.** Please explain briefly why there can be no general purpose method to convert any context-free grammar into an equivalent NFA.

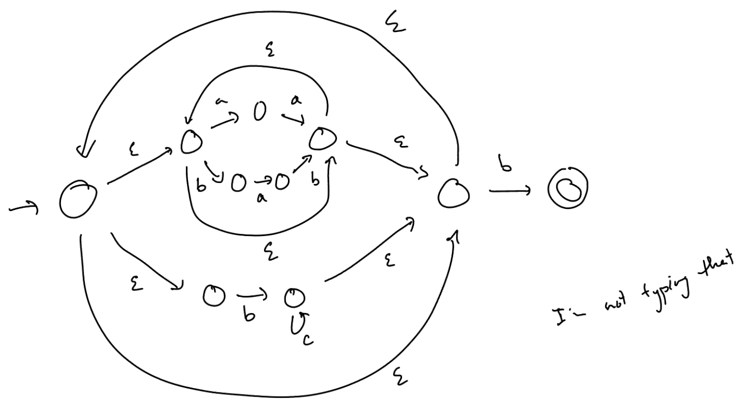
**Question 10.** Please give a stack machine for the language $\{a^n b^k a^n\}$ by converting a grammar.

**Question 11.** Please make a stack machine for the language $\{a^n b^k a^n\}$ "by hand" (not converting from a grammar).

# Answers!

1. a) $a^*b^*$
   b) $(ab+b)^*$
   c) Many ways to write it: $(b+c)^*(a+b)^*$, or maybe $(b+c)^*a(a+b)^* + (b+c)^*$
   d) $(cac+b+c)^*$

2.




3. a) $S \to abSa \mid \varepsilon$
   b) $S \to aSb \mid \varepsilon \mid b^2$
   c) $S \to ABC$
   $A \to aA \mid \varepsilon$
   $B \to bB \mid \varepsilon$
   $C \to cC \mid \varepsilon$
   d) $S \to aaSb \mid bT$
   $T \to aT \mid \varepsilon$
   e) All these are context free, which just means that the left side of the arrow is always just a single nonterminal symbol.
   f) $\{a^n b^k c^m\}$ is regular, with regular expression $a^*b^*c^*$.

4. a) $S \to SS \mid [X] \mid \varepsilon$
   $X \to xX \mid \varepsilon$
   b) $([a^*])^*$

5. Many possible answers, but I'll go with $L = \{a^n\}$. Then $\frac{d}{da}L = L$. (The simplest and overly-stupid answer would be $L = \emptyset$.)

6. Let's take $\frac{d}{da^i}L$. If some string in $L$ begins with $a^i$, then it must have $x = a^i y$ for some $y$, so the format of the whole string looks like $xax^2 = a^i yaa^i ya^i y$. So when we take the derivative, we get:

$$\frac{d}{da^i}L = \{yaa^i ya^i y \mid y \in \{a,b\}^*\}.$$

These are all different for different $i$, so $L$ has infinitely many derivatives, so $L$ is not regular.

7. a) $(aabbbb, S) \mapsto (abbbb, SXX) \mapsto (bbbb, SXXXX) \mapsto (bbbb, XXXX) \mapsto (bbb, XXX) \mapsto (bb, XX) \mapsto (b, X) \mapsto (\varepsilon, \varepsilon)$

   b) There are two ways to attempt to read $ab$, but both fail:
   $(ab, S) \mapsto_3 (ab, \varepsilon)$ stuck!
   $(ab, S) \mapsto_1 (b, SXX) \mapsto (b, XX) \mapsto (\varepsilon, X)$ stuck!

   c) $S \to aSXX \mid \varepsilon$
   $X \to b$

   d) $\{a^n b^{2n}\}$

   e) This makes one extra $X$, which will need to be canceled by one extra $b$, so it'll be $\{a^n b^{2n+1}\}$.

   f) This rule would make it impossible to ever clear an $X$ off of the stack. Then the only string accepted would be the empty string (after doing step 3 once). So the language would be $\{\varepsilon\}$.

8. Let $D_i = \frac{d}{db^i} L = \{b^{n-i} a^n\}$. These are all different for various $i$, so $L$ has infinitely many derivatives, so $L$ is not regular.

9. Some CFGs generate nonregular languages. (Like $S \to aSb \mid \varepsilon$.) So those ones are impossible to convert into an equivalent NFA.

10. I used this grammar:

    $S \to aSa \mid B$

    $B \to bB \mid \varepsilon$

    And now convert to a stack machine:

| read | pop | push |
|---|---|---|
| $\varepsilon$ | $S$ | $aSa$ |
| $\varepsilon$ | $S$ | $B$ |
| $\varepsilon$ | $B$ | $bB$ |
| $\varepsilon$ | $B$ | $\varepsilon$ |
| $a$ | $a$ | $\varepsilon$ |
| $b$ | $b$ | $\varepsilon$ |

11. .

| read | pop | push |
|---|---|---|
| $a$ | $S$ | $SX$ |
| $\varepsilon$ | $S$ | $T$ |
| $b$ | $T$ | $T$ |
| $\varepsilon$ | $T$ | $\varepsilon$ |
| $a$ | $X$ | $a$ |