Chapter 2: 3ab, 6c,

Chapter 3: 10

extra problems #1

Chapter 2 #3

a

$\rightarrow \bigcirc \underset{a,b}{\overset{a,b}{\rightleftarrows}} \circledcirc$ $\{ x \in \{a,b\}^+ \mid |x| \text{ is odd} \}$

b
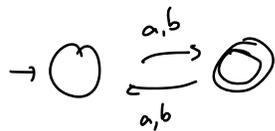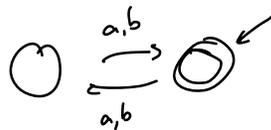
$\bigcirc \underset{a,b}{\overset{a,b}{\rightleftarrows}} \circledcirc$ $\{ x \in \{a,b\}^+ \mid |x| \text{ is even} \}$

Chapter 2 #6c
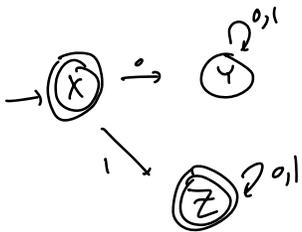
$M = \left(\{X, Y, Z\}, \{0, 1\}, \delta, X, \{X, Z\} \right)$

where:
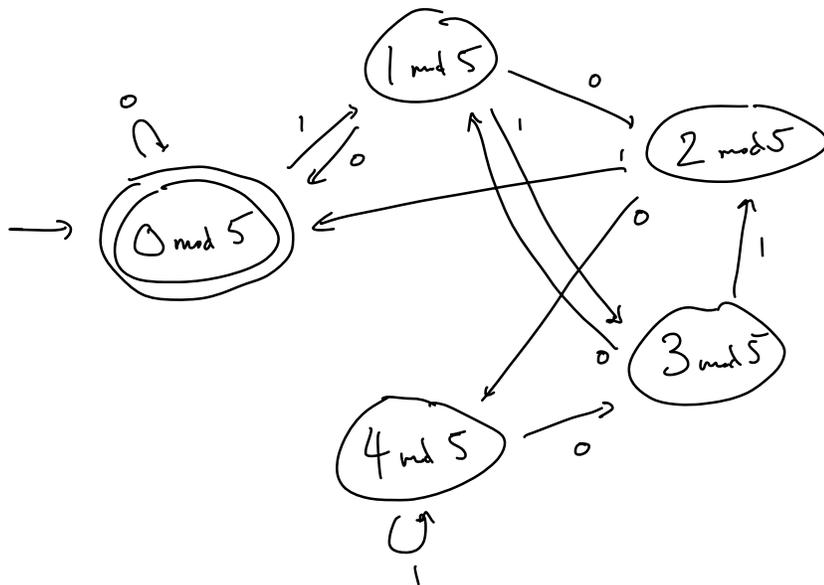
$\delta(X, 0) = Y$ $\delta(X, 1) = Z$

$\delta(Y, 0) = Y$ $\delta(Y, 1) = Y$

$\delta(Z, 0) = Z$ $\delta(Z, 1) = Z$

Chapter 3 #10   Binary #s div. by 5.

Lots of little algebra to find these arrows.
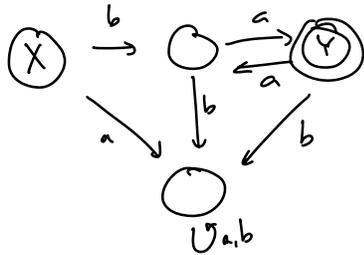
For example from $\boxed{2 \bmod 5}$

0 arrow: $\quad x = 5k+2 \rightarrow 2x = 2(5k+2) = 10k + 4 = 4 \bmod 5$

1 arrow: $\qquad\qquad \hookrightarrow 2x+1 = 2(5k+2)+1 = 10k + 5 = 0 \bmod 5$

etc.

Extra #1

Thm $\quad \delta^*(X, ba^{2n+1}) = Y$

PF Induction on n.

  Base case $n=0$ WB $\quad \delta^*(X, ba) = Y$ which is true.

  Inductive case Assume $\delta^*(X, ba^{2k+1}) = Y$, WB $\delta^*(X, ba^{2(k+1)+1}) = Y$.

  We have:
  $$\delta^*(X, ba^{2(k+1)+1}) = \delta^*(X, ba^{2k+3})$$
  $$= \delta^*(X, ba^{2k+1} a^2)$$
  $$= \delta^*(\delta^*(X, ba^{2k+1}), a^2)$$
  $$\overset{IH}{=} \delta^*(Y, a^2) = Y \qquad \text{Shown!}$$