Homework #10

Question 1. For each of these languages, make a Turing machine:

a) $\{x \in \{a,b\}^* \mid$ the first and last letter of x are the same $\}$.

b) $\{b^{2n}a^n\}$

c) $\{a^nba^n\}$

d) $\{x \in \{a,b\}^* \mid x$ uses the same number of as and bs$\}$

e) $\{xcycx \mid x, y \in \{a,b\}^*\}$

Question 2. In each part, give a Turing machine which computes the given function.

a) The "head" function, which takes a nonempty string on the alphabet $\{a,b\}^*$ and returns just its first letter.

b) The "tail" function, which takes a nonempty string on the alphabet $\{a,b\}^*$ and returns the whole thing except the first letter.

c) For $x \in \{a,b\}^*$, the function $f(x) = a^{|x|}$.

d) $f(a^n) = a^{2n}$. (One possible strategy you could use: first turn the string a^n into something like $\bar{a}^n b^n$ by marking each a and writing b at the end to match it. Then run across the whole string and turn everything into an a, resulting in a^{2n}.)

e) $f(a^{2n}) = a^n$. (You can use a similar strategy: mark an a at the beginning, and then blank one out at the end. Then when you're done, change every $\bar{a}$ back to an a.)

f) For binary numbers, $f(x) = x + 4$. (You can assume that x is large enough so that it already uses at least 3 digits.)

g) For binary numbers, $f(x) = x \div 4$. (Give only the quotient as the answer, ignoring any remainders.)

h) For binary numbers, $f(x) = x \mod 4$.