

MA451 Examples from Section 2.6 - 2.8
 Conditional Probability, Bayes' Rule and Law of Total Probability

1.) A couple has two children. List the sample space of possible outcomes. $\{G_1, G_2, B_1, G_2, G_1, B_2, B_1, G_2\}$
 What is the probability that both children are girls given that the oldest child is a girl?
 $P(G_1, G_2 | G_1) = P(G_1, G_2) / P(G_1) = 1/4 / 1/2 = 1/2$

2.) The probability that a woman lives long enough to celebrate her 70th birthday is 60% and the probability that a woman reaches her 80th birthday is 20%. If a woman reaches her 70th birthday, what is the probability that she will celebrate her 80th?

$$P(B|A) = P(A \cap B) / P(A) = P(B) / P(A) = 0.20 / 0.6 = 1/3 = .33$$

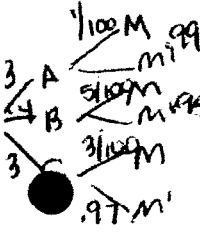
3.) The weather forecast says there is a 25% chance of rain on Saturday and a 25% chance of rain on Sunday

a.) Is the probability that it will rain on both days of the weekend .0625? Why or why not? $P(A \cap B) = P(A)$ not likely incl

b.) Is the probability that it will rain at some point during the weekend .50? Why or why not?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4.) The eBook Factory Warehouse employs three stock clerks Andy, Betty and Chris, who pull books off of shelves to fill customers' orders. Andy makes a mistake in an order one time per one hundred orders, Betty makes a mistake 5 times in a hundred orders and Chris makes a mistake 3 times per one hundred orders. Andy, Betty and Chris fill, respectively, 30%, 40% and 30% of all orders



a.) Summarize the information pictorially with a tree diagram.

b.) What is the probability that a mistake will be made in an order? $P(M) = P(A)P(M|A) + P(B)P(M|B) + P(C)P(M|C)$

c.) What is the probability that if a mistake is made in an order, the order was filled by Betty?

$$P(B|M) = P(B \cap M) / P(M) = \frac{.4(.05)}{.032} = .625$$

d.) What is the probability that if a mistake is made in an order, the order was filled by Andy?

$$P(A|M) = P(A \cap M) / P(M) = \frac{.3(.01)}{.032} = .094$$

5.) A study showed the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

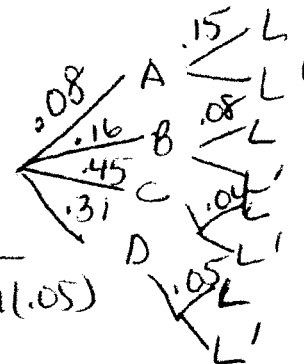
Type of driver	Percentage of all Drivers	Prob. of at least one Collision
Teen A	8%	.15
Young Adult B	16%	.08
Midlife C	45%	.04
Senior D	31%	.05

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult?

$$P(B|L) = \frac{P(B \cap L)}{P(L)}$$

$$= \frac{.16(.08)}{.08(.15) + .16(.08) + .45(.04) + .31(.05)}$$

$$= \frac{.0128}{.0538} = .238$$



6.) In a T-maze, a rat is given food if it turns left and a mild electric shock if it turns right. On the first trial, there is a 50-50 chance that that rat will turn either way. If the rat receives food on the first trial, the probability is .68 that it will turn left on the next trial. If the rat receives a shock on the first trial, the probability is .84 that it will turn left on the next trial.

- What is the probability that the rat will turn left on the second trial?
- Given that the mouse turned left on the second trial, what is the probability that it turned left on the first trial?

7.) It is known from experience that 60% of all labor-management disputes are over wages, 15% are over working conditions and 25% are over benefits. Also, 45% of wage disputes are resolved without strikes, 70% of the disputes over working conditions are resolved without strikes and 40% of the disputes over benefits are resolved without strikes.

- What is the probability that a labor-management dispute in this industry will be resolved without a strike?
- Given that a labor-management dispute did not result in a strike, what is the probability that it was a dispute over benefits?

6)

$$P(L_2) = P(L_1, A_2) + P(R_1, A_2)$$

$$= .5(.68) + .5(.84)$$

$$= .76$$

$$P(L_1 | L_2) = \frac{P(L_1, A_2)}{P(L_2)} = \frac{.5(.68)}{.76}$$

$$= .45$$

7.)

$$P(S') = .6(.45) + .15(.2) + .25(.4)$$

$$= .475$$

$$P(B | S') = \frac{.25(.4)}{.475} = \frac{.1}{.475} = .21$$