Practice Problems for Exam 1

Calculus I, MATH 1141

Fall 2020

Read each question carefully and show all work for full credit.

1. Approximate the slope of $y = e^x$ at $x = 0$ by filling in the table below, which shows the slope m_{PQ} of the secant line from $P = (0, 1)$ to $Q = (x, e^x)$.

x	m_{PQ}
0.5	
0.1	
0.01	
0	–
−0.01	
−0.1	
−0.5	

2. Evaluate the following limits using the limit laws, or state why the limits do not exist. If a limit is infinite, state whether the limit is $+\infty$ or $-\infty$.

a) $\lim_{x\to 3} \ln(x) - 4$

b) $\lim_{x\to -2} \dfrac{x^2 - 4}{x^2 + 5x + 6}$

c) $\lim_{x\to 0^+} \dfrac{3}{2x^3}$

d) $\lim_{x\to 0^-} \dfrac{3}{2x^3}$

e) $\lim_{x\to \infty} \dfrac{4x + 3}{2 - x}$

3. Find all horizontal and vertical asymptotes of the function $f(x) = \dfrac{\sqrt{x^4 + 1}}{x^2 - 9}$. Show your work with limits explicitly to support your answers.

4. Show that the equation $e^x = x^2$ has a solution in the interval $[-1, 0]$. Can you show it has a solution in an interval half that size?

5. a) State the limit definition of the derivative.

b) Find $f'(x)$ for $f(x) = 3x - x^2$ using your definition from part (a).

c) Find the equation of the tangent line to f at $x = 1$.

6. Let $f(x) = \begin{cases} x^2 - 2x, & x \le -1 \\ a - x^2, & x > -1 \end{cases}$

Find the value of a which makes f continuous at $x = -1$.

Show the criteria for continuity and your work with limits explicitly.

7. Let $f(x) = \ln(x)$. It is given that $\lim_{x\to 1} \ln(x) = 0$.

Let $\varepsilon = 0.1$. Find the value of $\delta > 0$, accurate to 4 decimal places, so that if $0 < |x-1| < \delta$, then $|\ln(x) - 0| < 0.1$.