Practice Problems for Exam 3

Calculus I, MATH 1141

Fall 2020

Read each question carefully and practice showing all work to earn full credit.

1. Find the absolute maximum and minimum values of the function on the given interval.
   a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$
   b) $f(x) = 1/x$ on the interval $[1, \infty)$
   c) $f(x) = \dfrac{e^x}{1 + x^2}$ on the interval $[0, 3]$
   d) $f(t) = 1 + \cos^2(t)$ on the interval $[\frac{\pi}{4}, \pi]$.

2. For each function, find the intervals on which f is increasing or decreasing. Find all local maximum and minimum values of f. Be sure to state both the critical numbers and the critical values.
   a) $f(x) = 6x^4 - 16x^3 + 1$
   b) $f(x) = 8x^{1/3} + x^{4/3}$
   c) $f(x) = \ln(2 + \sin(x)), \qquad 0 \le x \le 2\pi$

3. For each function, find the intervals on which f is concave up and concave down. Find all inflection points for f.
   a) $f(x) = e^{-x^2}$
   b) $f(x) = x^2 \ln(x)$, $x > 0$
   c) $f(x) = \dfrac{1}{x^2 + 1}$

4. Evaluate the following limits. Use L'Hospital's Rule where appropriate.
   a) $\lim\limits_{x \to \pi/4} \dfrac{\sin(x) - \cos(x)}{\tan(x) - 1}$
   b) $\lim\limits_{x \to 0} \dfrac{\tan(5x)}{\sin(3x)}$
   c) $\lim\limits_{t \to 0} \dfrac{e^{2t} - 1}{\sin(t)}$
   d) $\lim\limits_{\theta \to 0} \dfrac{1 + \cos(\theta)}{1 - \cos(\theta)}$
   e) $\lim\limits_{x \to \infty} (1 + \frac{2}{x})^x$

5. Find the general antiderivative of the following functions.

   a) $f(x) = x^3 - 5x^2 - 3x + 19$
   b) $g(t) = \dfrac{3}{t^2} - 8\sqrt{t}$
   c) $f(x) = 3e^x + \dfrac{2}{x}$
   d) $f(t) = \sin(t) - 5\sec^2(t)$
   e) $g(x) = 4\sinh(t) - 3\cosh(x)$
   f) $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$

6. Suppose $f''(x) = -2 + 12x - 12x^2$, $f(0) = 4$ and $f(1) = 6$. Find $f(x)$.

7. Suppose the acceleration of an object is given by $a(t) = 2t + 1$. Find the position $x(t)$ if $x(0) = 3$ and $v(0) = -2$.

8. Approximate the area under the curve $f(x) = \sin^2(x)$ from $x = 0$ to $x = \pi$ using 4 rectangles with function values evaluated at:

a) left endpoints; b) right endpoints; c) midpoints

Round your answers to 4 decimal places. Use your calculator to find the error in each approximation.

9. Express the following integral as the limit of Riemann sums and then evaluate the limit.

$\int_0^4 (x - x^2)\, dx.$

10. Use part one of the Fundamental Theorem of Calculus to find the derivative of each function.

a) $f(x) = \int_x^{2x} e^{t^2}\, dt$ b) $\int_1^{\sqrt{x}} \ln(1 + t^2)\, dt$

11. Use part two of the Fundamental Theorem of Calculus to evaluate the definite integrals.

a) $\int_1^8 x^{-2/3}\, dx$

b) $\int_1^3 \left(\frac{1}{t^2} + \frac{1}{t^3}\right) dt$

c) $\int_0^{\pi/2} \sin(\theta)\, d\theta$

d) $\int_0^{\pi/3} \sec(\theta)\tan(\theta)\, d\theta$

e) $\int_{-3}^3 e\, dx$

f) $\int_0^1 \left(e^x - \frac{4}{1 + x^2}\right) dx$