Important Definitions, Theorems and Suggested Problems for the Final Exam

MA 1141 Calculus I for Chemistry, Engineering and Physics
Fall 2020

1. Key Definitions

Definition 1. (Definition of Limit) *The limit of $f(x)$ as x approaches a is L if for all $\varepsilon > 0$, there exists $\delta > 0$ such that if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$. In this case, we write*

$$\lim_{x\to a} f(x) = L\,.$$

Definition 2. (Definition of Continuity) *A function $f(x)$ is continuous at $x = a$ if the following three conditions are satisfied:*

a) *$f(a)$ exists;*
b) *$\lim_{x\to a} f(x)$ exists;*
c) *$\lim_{x\to a} f(x) = f(a)$.*

Definition 3. (Definition of Derivative) *The derivative of $f(x)$ at $x = a$ is defined as the value of the limit,*

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}\,,$$

if the limit exists. Alternatively, the derivative of $f(x)$ is defined by,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}\,.$$

Definition 4. (Definition of Definite Integral) *Let f be a continuous function on an interval $[a, b]$. For $n > 0$, subdivide $[a, b]$ into n intervals of width $\Delta x = \frac{b-a}{n}$, with endpoints $x_0 = a$, $x_1 = a + \Delta x, \ldots, x_i = a + i\Delta x, \ldots, x_n = b$.*

The definite integral of f on $[a, b]$ is defined as the value of the limit,

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x\,.$$

Note: The sum $\sum_{i=1}^{n} f(x_i)\Delta$ is called a **Riemann Sum** of f. The sum above evaluates f at the right end point of each interval. We could also use the left endpoints, $\sum_{i=0}^{n-1} f(x_i)\Delta x$, or any point x_i^* in the ith interval, $\sum_{i=1}^{n} f(x_i^*)\Delta x$, and the value of the limit would be the same (as long as f is continuous).

2. Important Theorems

We have covered several important theorems this semester. Here are four whose statements are important to know.

Theorem 1. Intermediate Value Theorem. *Let f be a continuous function on a closed, bounded interval $[a, b]$. For every number k between $f(a)$ and $f(b)$, there exists $c \in (a, b)$ such that $f(c) = k$.*

Theorem 2. Mean Value Theorem. *Let f be a function that is continuous and differentiable on a closed, bounded interval $[a, b]$. There exists $c \in (a, b)$ such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}\,.$$

Theorem 3. Fundamental Theorem of Calculus, Part I. *Suppose f is continuous on $[a, b]$. Then for any $x \in [a, b]$,*

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)\,.$$

Theorem 4. Fundamental Theorem of Calculus, Part II. *Suppose f is continuous and that $F'(x) = f(x)$ for all x in $[a, b]$. Then*

$$\int_a^b f(x)\,dx = F(b) - F(a)\,.$$

3. Some Review Problems

In addition to the problem sets, practice problems and exams you have completed, a good source of review problems for the final exam are the Chapter Review sections at the end of each chapter. These problems require you to determine what technique to use to answer the question, which resembles the situation in an exam. In particular, I would recommend the following list of problems, which is not exhaustive, but is representative of the types of questions you may be asked.

Chapter 2 Concept Check: 2, 4, 5, 7, 12, 15
Review Exercises: 1, 3, 4, 8, 9, 25, 29, 35, 37

Chapter 3 Concept Check: 2, 7
Review Exercises: 1, 3, 4, 5, 8, 14, 21, 25, 27, 44, 61, 64, 70, 91, 107,

Chapter 4 Concept Check: 1, 4, 5
Review Exercises: 1, 2, 5, 7, 8, 12, 14, 45, 65, 66, 67, 68, 69, 74

Chapter 5 Concept Check: 2, 4, 6, 7
Review Exercises: 2, 3, 5, 12, 13, 14, 15, 17, 18, 19, 20, 24, 26, 27, 29, 35, 39, 47, 51, 53, 56, 63, 64