

Extra Practice Problems For Final Exam

Applied Calculus II, MA 120

Spring 2019

Since the problems in the text do not incorporate trigonometric functions into many of the topics, I have written some problems for each topic we have covered this semester that seems to me lacking in the text. This is not a comprehensive list of all possible problems, but should be used in combination with the list of suggested review problems from the text.

1. Find dy/dx for each of the following curves using implicit differentiation.

a) $\sin y = xe^x$

b) $\tan(xy) = y^2 - 3x$

- c) Find the equation of the tangent line to $4\cos(y^2) = xe^y$ at the point $(4, 0)$.

2. Evaluate the following definite integrals.

a) $\int_0^\pi x \sin x \, dx$

b) $\int_0^{\pi/4} x \tan(x^2) \, dx$

3. Consider the integral $\int_0^\pi \sin(x^2) \, dx$.

a) Approximate the value of the integral using $n = 6$ rectangles with heights taken at the left endpoint of each rectangle.

b) Approximate the value of the integral using the Trapezoidal Rule with $n = 6$.

c) Approximate the value of the integral using Simpson's Rule with $n = 6$.

4. Suppose the velocity of a particle is given by the function $v(t) = t^2 \cos t$, where v is measured in feet per second. Find the average velocity of the particle from $t = 1$ to $t = 3$ seconds.

5. a) Find the area of the region bounded by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \pi/4$.

b) Find the area of the region bounded by the curves $y = \sin(2x)$ and $y = -\sin(2x)$ between $x = 0$ and $x = \pi$.

6. Evaluate the following improper integrals using limits or show that they diverge.

a) $\int_1^\infty \frac{\sin(1/x)}{x^2} \, dx$

b) $\int_0^1 \frac{1}{\tan x} \, dx$

7. State whether each infinite series converges or diverges. If it is a convergent geometric series, find its sum.

a) $\sum_{n=3}^\infty 2 \cdot 3^{-n}$

b) $\sum_{n=0}^\infty 2^n$

c) $\sum_{n=1}^\infty \frac{e^n}{n!}$

d) $\sum_{n=1}^\infty \frac{n^2}{2^n}$

(Hint: Remember the *Ratio Test* from class: Suppose $\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = r$. If $r < 1$, then the series converges; if $r > 1$, then the series diverges; if $r = 1$, then the test is inconclusive.)

8. Find the Taylor polynomials based at 0 up to degree 4 for the following functions.

a) $f(x) = \tan x$

b) $f(x) = \sin(2x)$

9. Use substitution to find the Taylor series based at 0 for the following functions.

a) $f(x) = \sin(x^2)$ b) $f(x) = \cos(\sqrt{x})$

c) Use your answer to part (a) to evaluate $\int_0^\pi \sin(x^2) dx$ as an infinite series. Approximate the value of this integral by adding up the first three terms of the integrated series. What is the maximum error in your approximation?

10. Evaluate the following limits.

a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ b) $\lim_{x \rightarrow \frac{\pi}{2}^-} (x - \frac{\pi}{2}) \tan x$ c) $\lim_{x \rightarrow 0} x \ln |\sin x|$