Problem Set 3
Real Analysis, MATH 5471
Due October 29, 2025

Do the following problems from the text, *Real Analysis*, 4th edition, by R. Bartle and D. Sherbert.

Section 4.1: 10, 15
Section 4.2: 2, 4
Section 4.3: 5, 11
Section 5.1: 4, 7
Section 5.2: 11, 14

In addition, answer the following questions.

1. Suppose we have n functions $f_1, \ldots, f_n$ with $f_i : \mathbb{R} \to \mathbb{R}$ and n numbers $L_1, \ldots, L_n \in \mathbb{R}$ such that $\lim_{x\to c} f_i(x) = L_i$ for $i = 1, \ldots, n$.
a) Prove by induction that

$$\lim_{x\to c} f_1(x) \cdot f_2(x) \cdots f_n(x) = L_1 \cdot L_2 \cdots L_n.$$

b) Conclude that if $\lim_{x\to c} f(x) = L$, then

$$\lim_{x\to c} (f(x))^n = L^n \qquad \text{for any } n \in \mathbb{N}.$$

2. a) Suppose

$$f(x) = \begin{cases} x^2 & \text{for } x \geq 1 \\ -x + k & \text{for } x < 1 \end{cases}.$$

Find the value of k which makes f continuous at $x = 1$.

b) Let $f(x) = \dfrac{x^2 - 7x + 12}{x - 3}$, $x \neq 3$. Is there a value for $f(3)$ which will make f continuous on all of $\mathbb{R}$?