

Problem Set 6
Real Analysis, MATH 5471
Due December 17, 2025

Do the following problems from the text, *Real Analysis*, 4th edition, by R. Bartle and D. Sherbert.

Section 7.3: 3, 16

Section 8.1: 22

Section 9.1: 8

Section 8.2: 9, 13

Section 9.2: 2

Section 9.3: 1

Section 9.4: 6a-d, 16

In addition, answer the following questions.

1. The equation $\ln x = x - 2$ has 2 solutions. Use Newton's method to find approximations to each which are correct to 4 decimal places. What happens if $x_1 = 1/2$ is the initial point?
2. a) Write down the Taylor series based at $a = 0$ for $f(x) = e^x$.
b) Use the series for part (a) to write a series representation of $g(x) = e^{-x^2}$. Does the sequence of partial sums for your series converge uniformly to g ? You may justify your answer by using facts about the series for f .
(Hint: Do not start taking derivatives of g , but rather notice that $f(-x^2) = g(x)$ to get the series quickly.)
c) Use the series from part (b) to get a series representation for the function $G(x) = \int_0^x e^{-t^2} dt$.
(Note: G is an antiderivative for g , which is impossible to obtain using the usual methods of Calculus.)
d) Use $G(x)$ to obtain a series representation for $\int_0^1 e^{-t^2} dt$.