ERRATUM FOR: EXPONENTIAL DECAY OF CORRELATIONS FOR FINITE HORIZON SINAI BILLIARD FLOWS

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We are grateful to Malo Jézéquel who incited us to clarify Lemma 3.2 and (W2). We thank Damien Thomine for pointing out the flaws in Lemma 7.5 and in the proof of Lemma 4.6, and for requiring necessary clarifications in the definition of the neutral norm. Fortunately, this only affects the paper very locally, and we explain here how to amend it.

1. Lemma 3.2 and (W2) — Definitions (3.10)–(3.11) and (4.3)

Lemma 3.2 does not show that there exists κ_0 such that if the curvature κ of W is bounded by κ_0 then the curvature of the iterated curve is bounded by κ_0. (The reason being that if κ tends to zero then the quantity B_ξ^2/B^6 does not go to zero.) The proof of Lemma 3.2 shows that $F_1 = B_\xi/B^3$ (where $B = d\omega/d\xi$, and $B_\xi = dB/d\xi$) is bounded for all times. So condition (W2) selecting those stable curves belonging to $\mathcal{W}^s$ should be replaced by the requirement that $F_1 < C_1$ for some C_1. (Since Lemma 3.2 gives $\kappa^2 < 1/4 + F_1^2$, it follows that all iterated curves have curvature $\kappa < \kappa_0$ for some κ_0. However $\mathcal{W}^s$ does not necessarily contain *all* stable curves with $\kappa < \kappa_0$ for some κ_0.)

The definitions (3.10) and (3.11) of $\mathcal{B}^0$ and $\mathcal{B}^0_\sim$ involve functions f which are only continuous. We emphasize that "the neutral norm of f is finite" then means that the derivative of f in the flow direction exists almost everywhere on each $W \in \mathcal{W}^s$, and its integral against ψ with $|\psi|_{C^\alpha(W)} \le 1$ is uniformly bounded. (For example, if f is a Cantor function in the flow direction, its neutral norm vanishes, while if $f \in C^0(\Omega_0) \setminus C^0_\sim$ then there are no Dirac contributions in the derivative.) Similarly, when proving (4.3) in Proposition 4.10 for $f \in C^0(\Omega_0) \cap \mathcal{B}^0$ with $f \notin C^0_\sim$, there are no Dirac contributions.

2. Neutral norm estimate in Lemma 4.6

Lemma 4.6 states that, for each $f \in \mathcal{B}$, we have $\lim_{t\downarrow 0} \|\mathcal{L}_t f - f\|_{\mathcal{B}} = 0$. The proof of the neutral norm estimate in this lemma for $f \in \mathcal{C}^2(\Omega_0) \cap \mathcal{C}^0_\sim$ should be corrected as follows:

Take $f \in \mathcal{C}^0_\sim \cap \mathcal{C}^2(\Omega_0)$. Then $\|\mathcal{L}_t f - f\|_s \le Ct|\nabla f \cdot \hat\eta|_{\mathcal{C}^0(\Omega_0)}$ and $\|\mathcal{L}_t f - f\|_u \le Ct|\nabla f \cdot \hat\eta|_{\mathcal{C}^1(\Omega_0)}$ hold, with the proofs as published, where $\hat\eta$ represents the flow direction. In particular (4.30) holds since $\mathcal{L}_t f$ is Lipschitz on Ω for all $t \ge 0$. (In the flow direction, distances are preserved so that the flow preserves Lipschitz smoothness in the flow direction, even across collisions.)

The estimate written for the neutral norm (proved using both (4.3) and (4.30)) holds if $W \in \mathcal{W}^s$ undergoes no collisions under Φ_{-s} for $0 \le s \le t$. Now fix $t < \tau_{\min}$. Then each $W \in \mathcal{W}^s$ can undergo at most one collision by time $-t$.

If $W \in \mathcal{W}^s$ undergoes a collision by time $-t$, then partition W into at most three connected pieces, at most two of which will make no collision by time $-t$, and one connected curve $W' \subset W$ which will undergo a collision in this time interval. On the two pieces which will not undergo a collision, we estimate $\mathcal{L}_t f - f$ as before.

To estimate $\mathcal{L}_t f - f$ on W', we cannot use (4.3) as for the collisionless pieces, because of derivatives of jump discontinuities. Instead, we first note that $|W'| \le C\sqrt{t}$ due to the strict

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convexity of scatterers and the opposing convexity of W. Next, for $\psi \in \mathcal{C}^\alpha(W)$ with $|\psi|_{\mathcal{C}^\alpha(W)} \le 1$, we estimate using (4.30),

$$\begin{aligned}\left|\int_{W'} \partial_r\big((\mathcal{L}_t f - f)\circ \Phi_r\big)|_{r=0}\, \psi \, dm_W\right| &\le \left|\int_{W'} \mathcal{L}_t(\nabla f \cdot \hat\eta)\, \psi\, dm_W\right| + \left|\int_{W'} \nabla f \cdot \hat\eta\, \psi\, dm_W\right| \\ &\le |W'|^{1/q}\|\mathcal{L}_t(\nabla f\cdot\hat\eta)\|_s + |W'|^{1/q}\|\nabla f\cdot\hat\eta\|_s \;\le\; C t^{1/(2q)} |\nabla f\cdot \hat\eta|_{\mathcal{C}^0(\Omega_0)}\,.\end{aligned}$$

This, together with the previous bounds, implies that $\|\mathcal{L}_t f - f\|_{\mathcal{B}} \le C t^{1/(2q)} |\nabla f \cdot \hat\eta|_{\mathcal{C}^1(\Omega_0)}$, which proves Lemma 4.6 for $f \in \mathcal{C}^0_\sim \cap \mathcal{C}^2(\Omega_0)$. The approximation argument for general $f \in \mathcal{B}$ follows as in the published proof of Lemma 4.6.

3. Lemma 7.5 (and Lemma 3.9) and Theorem 1.2

The first inequality in the fourth line of the proof of Lemma 7.5 is wrong, so that the neutral estimate there is flawed. As a consequence, we must replace the statement $C^1(\Omega_0) \subset \mathcal{B}$ of Lemma 7.5 (and the corresponding inclusion in Lemma 3.9, also mentioned after Definition 2.12) by the weaker inclusion

$$C^1(\Omega_0) \cap C^0(\Omega) \subset \mathcal{B}\,.$$

The original Lemma 7.5 is used in the paragraph before (5.1) to deduce that the domain of X contains $C^2(\Omega_0)$. This consequence of the original Lemma 7.5 must be replaced by the observation that if $f \in C^2(\Omega_0) \cap C^0(\Omega)$ (so that $\nabla f \in C^1(\Omega_0)$) is such that $(\nabla f)\cdot\hat\eta$ belongs to $C^0(\Omega)$, then the corrected version of Lemma 7.5 implies that f is in the domain of X, i.e. $Xf \in \mathcal{B}$.

This consequence of the original Lemma 7.5 is also used in the statement and proof of Theorem 1.2, which must be modified accordingly. In the statement of Theorem 1.2, in addition to requiring that $f \in C^2(\Omega_0) \cap C^0(\Omega)$, we add the requirement that $(\nabla f)\cdot\hat\eta \in C^0(\Omega)$. Then, in the proof of Theorem 1.2 in Section 9.1, we replace the claim that Lemma 7.5 implies that $C^2(\Omega_0) \cap C^0(\Omega) \subset \mathrm{Dom}(X)$ by the same observation as above: if $f \in C^2(\Omega_0) \cap C^0(\Omega)$ and $(\nabla f)\cdot\hat\eta \in C^0(\Omega)$, then $f \in \mathrm{Dom}(X)$. The estimates then hold as written.