Elementary Theory of Groups and Group Rings

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Abstracts

Incorporating Examples of Remarkable Groups into your Teaching

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When a student is first introduced to group theory, the common groups provided as examples are infinite and finite cyclic groups, dihedral symmetry groups, and permutation groups. It can be challenging to provide further interesting infinite groups that can be explored and understood in some detail. In this talk, we discuss how to introduce Thompson's group F, the lamplighter group, self-similar groups and Baumslag-Solitar groups to your students and the benefits of doing so. Our work is dedicated to Gilbert Baumslag and is inspired by his approachable teaching style and his enthusiasm for guiding the next generation of group theorists.

Some Properties of the Baumslag Groups $G(m,n)$

Anthony Clement
CUNY

The group $G = \langle a, b | a = [a, a^b] \rangle$ re-expressed as

$$G(1,2) = \langle a, b | b^{-1}a^{-1}bab^{-1}ab = a^2 \rangle$$

first appeared in G. Baumslag's 1969 paper "*A non-cyclic one-relator group all of whose finite quotients are cyclic*" in which he showed that every finite quotient of $G = \langle a, b | a = [a, a^b] \rangle$ is cyclic and as a result presented at the time yet another example of a one-relator group which was not residually finite. In this talk I will describe the structure and some properties of the Baumslag groups $G(m,n) = \langle a, b | b^{-1}a^{-1}ba^m b^{-1}ab = a^n \rangle$.

The Elementary Theory of Free Inverse Monoids: About Decidable and Undecidable Fragments

Volker Diekert

University of Stuttgart

In a joint paper with Florent Martin, Géraud Sénizergues, and Pedro V. Silva we showed in 2016 that the problem to solve equations with idempotent variables in free inverse monoids is EXPTIMEcomplete. The notion of idempotent variables is crucial since a classical Result of Rozenblatt states that the set of solvable equations in a free inverse monoid is undecidable, in general. In my talk I will report on some ongoing work with Géraud Sénizergues which shows that the elementary theory in free inverse monoids with idempotent variables is undecidable. The problem whether the existential theory of free inverse monoids with idempotent variables is decidable remains open.

Solving the Conjugacy Decision Problem via Machine Learning

Jonathan Gryak[1], Robert M. Haralick[2], Delaram Kahrobaei[2]

[1]University of Michigan, [2]CUNY

Machine learning and pattern recognition techniques have been successfully applied to algorithmic problems in free groups. In this talk, we seek to extend these techniques to finitely presented non-free groups, with a particular emphasis on polycyclic and metabelian groups that are of interest to non-commutative cryptography.

As a prototypical example, we utilize supervised learning methods to construct classifiers that can solve the conjugacy decision problem, i.e., determine whether or not a pair of elements from a specified group are conjugate. The accuracies of classifiers created using decision trees, random forests, and N-tuple neural network models are evaluated for several non-free groups. The very high accuracy of these classifiers suggests an underlying mathematical relationship with respect to conjugacy in the tested groups.

Rational Subgroups and Invariants of F_4 and G_2

Neha Hooda

Fairfield University

Over an algebraically closed field the classification of semisimple algebraic groups is well understood. Two semisimple linear algebraic groups are isomorphic if and only if they have isomorphic root data (Chevalley Classification Theorem). Moreover, corresponding to any subdiagram of the Dynkin diagram of the group, there exists a subgroup which realizes it. But this fails to hold for an non-algebraically closed field. Thus over an non-algebraically closed field k, it is important to know what are all the simple k-subgroups of a given group. We try to answer this for exceptional algebraic groups of type F_4 and G_2.

Attracting Trees of “Random” Automorphisms of Free Groups

Ilya Kapovich
University of Illinois, Urbana-Champaign

For a free group automorphism the property of being fully irreducible provides a counterpart of a pseudo-Anosov homeomorphism of a surface. It has been known, for general geometric reasons, that “random” or “generic” (in the sense of being obtained by a long random walk) elements of $Out(F_r)$ are fully irreducible. We establish finer structural properties of such “random” fully irreducibles $\phi \in Out(F_r)$ (where $r \geq 3$) and prove that generically such an element ϕ is “ageometric”, with the ideal Whitehead graph of ϕ being a disjoint union of triangles. In particular this result implies that the attracting tree of ϕ is trivalent, that is all of its branch points have valency 3. The talk is based on a joint paper with Joseph Maher, Catherine Pfaff and Samuel Taylor.

First-Order Properties of Group Rings of Limit Groups

Olga Kharlampovich
Hunter College, CUNY

We will discuss definable sets in group rings of limit groups and give elementary classification of them. These are joint results with A. Miasnikov.

Gröbner Basis Methods in Group Theory

Martin Kreuzer
University of Passau

After introducing non-commutative (and commutative) Gröbner bases, we point out some examples of their ubiquity and usefulness for theoretical arguments. Then we proceed to look at some applications of the computation of non-commutative Gröbner bases, for instance checking finiteness of finitely peresented groups and calculating invariants such as the Hilbert-Dehn function. Surprisingly, also commutative Gröbner bases have found some usage for non-commutative groups, e.g., the decomposition of finite Z-algebras via their center or calculations involving the Burnside ring. The talk finished with a look at the current state of implementing algorithms for the computaiton of non-commutative Gröbner bases and a brief look at some open problems for which Gröbner bases could possibly be useful.

On Elementary Theories of One-Relator Groups

Alexei Miasnikov
Stevens Institute of Technology

One-relator groups are among the most studied and yet very intriguing objects in modern group theory. Recent breakthroughs in model theory of free and hyperbolic groups shed some light on elementary theories of one-relator groups. In this talk I will discuss some new results and open problems in this area. Based on a joint work with Olga Kharlampovich.

Infinite Nested Radicals

Barry Mittag
Western Connecticut State University

We prove that each positive integer n can be represented as an integer infinite nested radical and provide an easy method to determine the convergence. The method is straightforward and involves looking at integral quadratic forms. After we proved this we discovered that the result appears in Ramanujan's Lost Notebook. However our proof is so straightforward and the result so striking we think it should be publicized.

Applications of Leighton's Theorem

Walter D. Neumann
Columbia University

"Leighton's theorem" states that two finite connected graphs which have isomorphic universal covers have isomorphic finite covers. Leighton's theorem was later extended to colored graphs.

In a paper of Jason Behrstock and myself, *"Quasi-isometric classification of non-geometric 3-manifold groups"* (J. reine angew. Math. 2012, 101-120) we used a paper of mine *"On Leighton's graph covering theorem"* (Groups, Geometry, and Dynamics 4 2011, 863–872), to prove *under strong conditions* that any two irreducible non-geometric 3-manifolds with quasi-isometric fundamental groups have commensurable fundamental groups. We still wonder to what extent the "strong conditions" are needed. It depends in part to what extent a refined version of Leighton's Theorem is still valid.

The refined version of Leighton adds an additional type of "color" at each vertex—a finite group acting on the set of edges at the vertex. The refined Leighton is valid if its enhanced "graph of colors" is a tree (which is what allows the partial results for 3-manifold groups).

In the meantime, others have been wondering about essentially the same question. For example, Tullia Dymarz, Jen Taback and Kevin Whyte are looking at a related question for the Baumslag-Gersten group and related groups.

On Vector-Valued Hecke Forms

Gerhard Rosenberger
University of Hamburg

We discuss a general theory of vector-valued Hecke forms associated to finite-dimensional representations of the Hecke matrix group.

On Conditions for Polynomial Growth in Inverse Semigroups

Lev Shneerson
Hunter College, CUNY

We study finite presentations of inverse semigroups having polynomial growth and the connection between the growth and Gelfand-Kirillov dimension in some classes of finitely generated inverse semigroups.
Part of the talk will be based on joint work with David Easdown and Stuart Margolis.

Tropical Cryptography

Vladimir Shpilrain
CUNY

We employ tropical algebras as platforms for several cryptographic schemes by mimicking some well-known "classical" schemes in the "tropical" setting. What it means is that we replace the usual operations of addition and multiplication by the operations $\min(x, y)$ and $x + y$, respectively. An obvious advantage of using tropical algebras as platforms is unparalleled efficiency because in tropical schemes, one does not have to perform any multiplications of numbers since tropical multiplication is the usual addition.

Bi-Interpretability of the Ring of integers $\mathbb{Z}$ with the Pure Groups $SL(n, \mathbb{Z})$, $GL(n, \mathbb{Z})$ and $T(n, \mathbb{Z}), n > 2$.

Mahmood Sohrabi
Stevens Institute of Technology

In this talk I present some recent results on bi-interpretability of the ring of integers $\mathbb{Z}$ with the special linear group, $SL(n, \mathbb{Z})$, the general linear group, $GL(n, \mathbb{Z})$, and the group of invertible upper triangular matrices $T(n, \mathbb{Z})$, where $n > 2$. As a corollary we conclude that each of these groups is quasi finitely axiomatizable and prime. We note that the case of $T(n, \mathbb{Z})$ is specially interesting since by a result of A. Khelif the group of upper unitriangular matrices $UT(n, \mathbb{Z})$ over integers is not bi-interpretable with $\mathbb{Z}$, while $T(n, \mathbb{Z})$ is a finite extension of $UT(n, \mathbb{Z})$. This is a joint work with Alexei Myasnikov.

Unitriangular Representations of a Subtheory of the Universal Theory of the Heisenberg Group

Anthony Gaglione[1], Dennis Spellman[2]
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Let us say that a group G is **commutative transitive (CT)** provided the centralizer of any nontrivial element $g \in G \backslash \{1\}$ is abelian. More generally, given an integer $n \geq 0$, we say that G is **commutative transitive of level** n or satisfies CT(n) provided the centralizer of any element $g \in G \backslash Z_n(G)$ outside the n-th term of the upper central series is abelian. These properties are captured by universal sentences. For exampe, CT(0) is captured by

$$\forall x, y, z\, (((y \neq 1) \wedge ([x, y] = 1) \wedge ([y, z] = 1)) \rightarrow ([x, z] = 1))$$

while CT(1) or NZCT **(noncentral commutative transitivity**) is captured by

$$\forall x, y, z, w\, ((([y, w] \neq 1) \wedge ([x, y] = 1) \wedge ([y, z] = 1)) \rightarrow ([x, z] = 1))\,.$$

A.G. Myasnikov and V.N. Remeslennikov proved that the universal theory $Th_\forall(F)$ of a rank 2 free group F in the first-order language with equality $L_0[F]$ appropriate for group theory and containing constants from F is axiomatizable by the set $Q(F)$ of quasi-identities of $L_0[F]$ true in F together with CT when the models are resticted to F-groups, i.e. groups containing a distinguished copy of F as a subgroup. We can remove the restriction on models by including the **diagram** of F in the axiomatization. Here $diag(F)$ is the set of all atomic and negated atomic sentences of $L_0[F]$ true in F. For us the significance of rank 2 here is it is the least rank free group F such that all higher rank free groups are universally equivalent to F in the language $L_0[F]$. Let $c \geq 2$ be an integer. In the case of the variety N_c of all groups nilpotent of class at most c this minimum rank is $\max\{2, c-1\}$.

We may paraphrase a question of A.G. Myasnikov as follows. Let $c \geq 2$ be an integer.Let G be free of rank $\max\{2, c-1\}$ in N_c. Is $Th_\forall(G)$ with respect to $L_0[G]$ axiomatizable by $diag(G) \cup Q(G) \cup \{\mathrm{CT}(c-1)\}$?

If R is a commutative ring with $1 \neq 0$, let $U(R)$ be the group of all $3x3$ upper unitriangular matrices with entries in R. The Heisenberg group H is $U(\mathbb{Z})$. H is free in N_2 on the generators

$$a_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and } a_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In the case $c = 2$, Myasnikov's question becomes: Is $Th_\forall(H)$ with respect to $L_0[H]$ axiomatizable by $diag(H) \cup Q(H) \cup \{NZCT\}$? Now the models of $diag(H) \cup Q(H)$ are precisely the H-groups G admitting an H-embedding $G \rightarrow U(R)$ for some locally residully-$\mathbb{Z}$ ring R. We have found:

(1) A sufficient condition on the representation for G to satisfy NZCT.

(2) A sufficient condition on the represnetastion for G to be a model of $Th_\forall(H)$.

Neither condition is necessary. None the less, our main result is that, in the case the H-embedding $G \rightarrow U(R)$ is surjective, G is a model of $Th_\forall(H)$ if and only if G is a model of $diag(H) \cup Q(H) \cup \{NZCT\}$.

Lamplighter Groups as Bireversible Automaton Groups

Ben Steinberg
CUNY

Grigorchuk and Zuk constructed in 2000 the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ as an automaton group and used it to perform spectral computations that led to a counterexample to the strong Atiyah conjecture.

Shortly afterward Pedro Silva and I gave a construction of $A \wr \mathbb{Z}$ as an automaton group for any finite abelian group A. This construction uses so-called reversible automata and used a representation as rational power series over finite rings.

In 2015 Bondarenko, D'Angeli and Rodaro realized the lamplighter group $\mathbb{Z}_3 \wr \mathbb{Z}$ as a bireversible automaton group. Previously known bireversible automaton groups tended to be non-amenable (often virtually free) or virtually nilpotent, so this was quite a surprise.

Bondarenko and Savchuk announced a construction of $A \wr \mathbb{Z}$ as a bireversible automaton group for A an elementary abelian p-group with p odd using rational power series over finite fields.

In this talk we discuss realizations of lamplighter groups $A \wr \mathbb{Z}$ with A finite abelian as rational series over finite rings. We show that bireversibility seems to impose a constraint on the 2-Sylow subgroups. This is joint work with Rachel Skipper (Göttingen).

Rewriting in Thompsons Group F

Zoran Sunic
Hofstra University

It is not known if Thompsons group F admits a finite confluent rewriting system. We construct a system that is not finite, "but it comes close." Namely, we construct a regular, bounded, prefix-rewriting system for F over its standard 2-generator set. Modulo the jargon, this means that one can rewrite any word to its normal form, and thus solve the word problem, by using a device with uniformly bounded amount of memory – in other words, even I can do it. Our system is based on the rewriting system and the corresponding normal form introduced by Victor Guba and Mark Sapir in 1997. Joint work with Nathan Corwin, Gili Golan, Susan Hermiller, and Ashley Johnson.

On Gilbert Baumslag and Symbolic Computation

Al Thaler
National Science Foundation

This talk describes Gilbert Baumslag's early interest in and work on symbolic computation (aka MAGNUS) and the National Science Foundation's role in supporting that endeavor.

Quadratic Equations in Free Metabelian Groups

Alexander Ushakov
Stevens Institute of Technology

We prove that the Diophantine problem for orientable quadratic equations in free metabelian groups is decidable and furthermore, NP-complete. In the case when the number of variables in the equation is bounded, the problem is decidable in polynomial time. This a joint work with I. Lysenok.
