

# INFINITE GROUP THEORY 2015

## Schedule

THURSDAY APRIL 23

FAIRFIELD UNIVERSITY

ROOM 101 - DIMENNA-NYSELIUS LIBRARY

- 9:30 - 10:00 Registration and Coffee
- 10:00 - 10:15 Welcome and Opening Remarks
- 10:15 - 11:00 Anthony Gaglione, USNA  
“The universal theory of free Burnside groups”
- 11:00 - 11:15 Coffee
- 11:15 - 12:00 Volker Diekert, University of Stuttgart  
“Amenability and Conjugacy”
- 12:00 - 1:30 Lunch (Off Campus)
- 1:30 - 2:15 Alexei Myasnikov, Stevens Institute  
“Model theory of free solvable groups”
- 2:20 - 3:00 Delaram Kahrobaei, CUNY  
“Conjugacy Problem in Polycyclic Groups”
- 3:05 - 3:50 Bob Gilman, Stevens Institute  
“Variations on the Word Problem”
- 4:00 - 4:40 Paul Baginski, Fairfield University  
“Definability and Nilpotence in Groups with  
Bounded Chains of Centralizers”
- 4:45 - 5:30 Ben Fine, Fairfield University  
“On CT and CSA Groups”
- 5:35 - 6:15 Reception in Kelley Center
- 6:00 Dinner in the Kelley Center Banquet Hall

FRIDAY APRIL 24

CUNY GRADUATE CENTER

MORNING SESSION - ROOM 9207

- 9:50 - 10:10 Registration and Coffee  
10:10 - 10:15 Welcome and Opening Remarks  
10:15 - 11:00 Martin Kreuzer, University of Passau  
"Computational Aspects of Burnside Rings"  
11:10 - 11:55 Vladimir Shpilrain, CUNY  
"Homomorphic encryption of group elements"  
12:00 - 12:45 Jane Gilman, Rutgers  
"The Non-Euclidean Euclidean Algorithm"  
12:50 - 2:20 Lunch (Off Campus)

AFTERNOON SESSION 1 - ROOM C415A

- 2:20 - 3:05 David Garber, Holon Institute of Technology  
"The Alternating Subgroup of the Colored  
Permutation Group"  
3:10 - 3:55 Olga Kharlampovich, CUNY  
"Tarski Problems for Algebras"  
3:55 - 4:15 Coffee

AFTERNOON SESSION 2 - ROOM 5417

- 4:15 - 5:00 Eugene Plotkin  
"Equations over algebras: The logical geometry"  
6:00 - Dinner out

# INFINITE GROUP THEORY 2015

## Abstracts

**Prof. Paul Baginski**

**Title:** Definability and Nilpotence in Groups with Bounded Chains of Centralizers

**Abstract:** A group  $G$  has bounded chains of centralizers ( $G$  is  $\mathfrak{M}_C$ ) if every chain of centralizers  $C_G(A_1) \leq C_G(A_2) \leq \dots$  is finite. While this class of groups is interesting in its own right, within the field of logic known as model theory,  $\mathfrak{M}_C$  groups have been examined because they strictly contain the class of stable groups. Stable groups, which have a rich literature in model theory, robustly extend ideas such as dimension and independence from algebraic groups to a wider setting, including free groups. Stable groups gain much of their strength through a chain condition known as the Baldwin-Saxl chain condition, which implies the  $\mathfrak{M}_C$  property as a special case.

Several basic, but key, properties of stable groups have been observed by Wagner and others to follow purely from the  $\mathfrak{M}_C$  condition (this builds upon the work of Bludov, Khukhro, and others). These properties were, as one would expect, purely group-theoretic. From the perspective of logic, the class of  $\mathfrak{M}_C$  groups should be unruly, since the  $\mathfrak{M}_C$  cannot be captured using first-order axioms (unless one insists on a fixed finite centralizer dimension). Yet we have uncovered that  $\mathfrak{M}_C$  groups possess a logical property of stable groups as well, namely the abundance of definable nilpotent subgroups. We shall present this result and describe the current investigations for finding an analogue for solvable subgroups.

Joint work with Tuna Altinel.

**Prof. Volker Diekert**

**Title:** Amenability and Conjugacy

**Abstract:** The talk is based on a joint work with Alexei Myasnikov and Armin Weiß. In a paper presented at LATIN 2104 we investigated the conjugacy problem in Baumslag's group  $BG(1,2) = \langle a, b \mid (bab^{-1})a(bab^{-1})^{-1} = a^2 \rangle$ ; and we proved that its conjugacy problem  $\mathbf{BG}_{1,2}$  is decidable in polynomial time in a strongly generic setting. In the present talk we take a more general viewpoint. We examine Schreier graphs of amalgamated products and HNN extensions. For an amalgamated product  $G = H \star_A K$  with  $[H : A] \geq [K : A] \geq 2$ , the Schreier graph with respect to  $H$  or  $K$  turns out to be non-amenable if and only if  $[H : A] \geq 3$ . Moreover, for an HNN extension of the form  $G = \text{Gen}H, bbab^{-1} = \phi(a), a \in A$ , we show that the Schreier graph of  $G$  with respect to the subgroup  $H$  is non-amenable if and only if  $A \neq H \neq \phi(A)$ .

As application of these characterizations we show that under certain conditions the conjugacy problem in fundamental groups of finite graphs of groups with free abelian vertex groups can be solved in polynomial time on a strongly generic set. Furthermore, the conjugacy problem in groups with more than one end can be solved with a strongly generic algorithm which has essentially the same time complexity as the word problem. These are rather striking results as the word problem might be easy, but the conjugacy problem might be even undecidable. Finally, our results yield another proof that the set where the conjugacy problem of the Baumslag's group  $BG(1,2)$  is decidable in polynomial time is also strongly generic. The results are from a paper which will appear at ISSAC 2015.

**Prof. Ben Fine**

**Title:** On CT and CSA Groups

**Abstract:** The concept of commutative transitivity (CT) and the related idea of CSA groups play an important role in the study of algebraic geometry over groups. In the presence of full residual freeness they are equivalent. CSA always implies CT but there exists both finite and infinite non-CSA CT groups. In this talk we give a complete characterization of CT non-CSA groups. This is joint work with A. Gaglione, G. Rosneberger and D. Spellman.

**Prof. Anthony Gaglione**

**Title:** The universal theory of free Burnside groups of large prime exponent

**Abstract:** We characterize the universal theory of the free Burnside groups of any fixed large prime exponent. We also introduce sufficient conditions for the free Burnside group of any fixed large prime exponent and of any rank  $r$ ,  $1 \leq r \leq s$ , to be universally equivalent to the free Burnside group of rank  $s$  in the language extended by adjoining names for the elements of the free rank  $r$  Burnside group  $B$  where  $B$  is taken as a relatively free factor. (This is joint with Dennis Spellman and Seymour Lipschutz.)

**Prof. David Garber**

**Title:** Centralizers of centralizers of parabolic subgroups of braid groups and its application to the conjugacy problem

**Abstract:** We characterize the centralizer of the centralizer of all parabolic subgroups of the braid groups. We apply this result to provide a new and potentially more efficient solution to the subgroup conjugacy problem for these parabolic subgroups. Based on joint work with Arkadius Kalka, Eran Liberman and Mina Teicher.

**Prof. Bob Gilman**

**Title:** Variations on the word problem.

**Abstract:** The word problem for finitely presented groups is one of the famous decision problems of combinatorial group theory, not least because for some groups it is recursively unsolvable. In this talk we discuss connections with complexity theory, in particular complexity cores and challenger solver games.

**Prof. Jane Gilman**

**Title:** The Non-Euclidean Euclidean Algorithm

**Abstract:** Let  $A$  and  $B$  be elements of  $\text{PSL}(2; \mathbb{R})$  with  $G = \langle A, B \rangle$  non-elementary. In a series of papers written in the 1970's, Rosenberger together with Purzitsky pioneered the concept of a trace minimizing procedure to determine the discreteness properties of  $G$ . Their work has been extended over the years to geometric and algebraic algorithms for discreteness and to some cases when  $G$  is a subgroup of  $\text{PSL}(2; \mathbb{C})$ . In this talk we give an interpretation of the Gilman-Maskit geometric form of the algorithm as a Non-Euclidean Euclidean Algorithm for the cases when (i)  $A$  and  $B$  are hyperbolics with disjoint axes and (ii) when one or both of  $A$  and  $B$  are allowed to be parabolic instead of hyperbolic.

**Prof. Delaram Kahrobaei**

**Title:** Conjugacy problem in Polycyclic groups

**Prof. Olga Kharlampovich**

**Title:** Tarski Problems for Algebras

**Abstract:** We have more new results about rings elementarily equivalent to a finitely generated free associative algebra over a field and about group rings. These are joint results with A. Miasnikov.

**Prof. Martin Kreuzer**

**Title:** Computational Aspects of Burnside Rings

**Abstract:** In this talk we reconsider Burnside Rings, a classical tool in Group Theory, from a computational perspective. Using modern computer algebra techniques, many results about these commutative rings that were proved in the second half of the last century can be transformed into effective algorithms. After recalling the definition and basic properties of the Burnside ring, we show how to compute an explicit presentation in terms of generators and relations, how to determine the prime ideal spectrum and its connected components, and how to relate the Burnside ring to its normalization by computing the conductor ideal and the idempotents. Moreover, we introduce explicit methods for computing classical maps between Burnside rings such as the restriction and inflation maps. All algorithms will be accompanied by actual computations of concrete examples.

**Prof. Alexei Myasnikov**

**Title:** Model Theory of Free Solvable Groups

**Prof. Eugene Plotkin**

**Title:** Equations over Algebras: The Logical Geometry

**Abstract:** Our aim is to describe a uniform approach to treat equations over algebras and over models. The main goal is to present a machinery which allows us to extend geometry of algebraic sets over algebras to geometry of definable sets.

**Prof. Vladimir Shpilrain**

**Title:** Homomorphic encryption of group elements