Problem Set 3: PRODUCTION THEORY

1.)

$$Q = L^{.5}K^{.5}$$

SHORT RUN PRODUCTION FUNCTION -- Suppose that the firm's capital base is fixed at K = 50 in the short run.

a) DERIVE an expression for the short run production function. Q = $\sqrt{50}\,\sqrt{L}$

b) FILL IN the table below:

L	Q = TP	AP	MP
0	0	—	—
25	35.35	1.41	1.41
50	50	1	0.59
75	61.24	0.82	0.45
100	70.71	0.71	0.38

Quantity = quantity of output

Labor = quantity of input

2.) Suppose the cost function is given by $\mathbf{C = 1200 + 2Q^2}$.

a) FILL IN the table below:

Q = TP	FC	VC	AFC	AVC	ATC	MC
0	1,200	0	—	—	—	-
50	1,200	5,000	24	100	124	100
70.71	1,200	10,000	16.97	141.43	158.4	241.43
86.60	1,200	15,000	15.86	173.21	187.03	314.64
100.00	1,200	20,000	12	200	212	373.13

b. The equation for FC = 1200 AFC = $\frac{1200}{Q}$

c. The equation for VC = $2Q^2$ AVC = $2Q$

d. The equation for MC = $4Q$

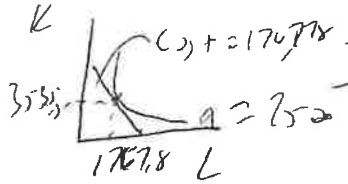
For the equations include any variables where necessary

LONG RUN – SUPPOSE CAPITAL AND LABOR ARE BOTH VARIABLE!

a. If the firm needs to produce **2500 units of output,** and the price **of labor and capital are $50 and $25** respectively, what are the cost minimizing levels of input usage? Plot this on a diagram. How much must the firm spend to generate these 2500 units?

$MPL = \frac{1}{2}\sqrt{\frac{K}{L}}$ $\quad MPK = \frac{1}{2}\sqrt{\frac{L}{K}}$ $\quad 2500 = \sqrt{L \cdot 2L}$

$= \sqrt{2}\,L$



$\frac{\frac{1}{2}\sqrt{\frac{K}{L}}}{\frac{1}{2}\sqrt{\frac{L}{K}}} = \frac{50}{25} \Rightarrow \frac{K}{L} = \frac{2}{1} \quad K = 2L$

$L = 1767.8$

$K = 3535.5$

$C = (1767.8)(50) + (3535.5)(25)$

$= \$176,778$

b. Are returns to scale increasing, decreasing, or constant with this production function?

Returns to scale = $Q = \sqrt{KL}$

Double K, L

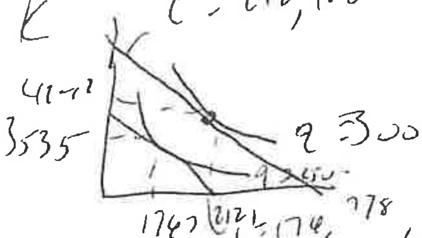
$Q = \sqrt{2K \cdot 2L} = \sqrt{4KL} = 2\sqrt{KL}$

Constant RTS

c. Suppose the demand for this product increases because health advantages for consumers are discovered. If the firm increases **output to 3000,** what is the new optimal quantity of inputs to use? Draw an expansion path in your diagram above.

Expansion path = $3000 = \sqrt{2}\,L$

$C = 212,100 \qquad L = 2121 \qquad K = 4242$



d. Suppose the firm acquired new production technology such that the production function becomes:

Q = 10L + 90K

Given constant input prices, what is the firm's new optimal quantity of inputs to use to produce **3000 units** of output? Show in a diagram and explain.

$MPL = 10 \quad MPK = 90$

$\frac{10}{90} \neq \frac{50}{25} \qquad \frac{1}{9} \neq \frac{2}{1}$ use all capital

$90K = 3000$

$K = 33.\overline{3}$

3) Suppose that you are a firm that produces xylophones. You have a production technology to produce xylophones that can be written as:

$$y = k^{1/2} l^{1/2}$$

Where k represents the units of capital employed at your production facility, l is the number of labor hours employed and y is your total production of xylophones. Assume that labor costs $10 per hour and that capital costs $250 per unit.

a) Suppose that you are currently employing 100 units of capital. If you have expected sales equal to 1,000. Calculate your optimal choice of labor

$y = 10L^{\frac{1}{2}}$

$1000 = 10\sqrt{L}$

$100 = \sqrt{L}$

$L = 10{,}000$

b) Given your answer to (a), calculate your marginal and average cost of production.

$MC = \frac{w}{MP_L} \Rightarrow \frac{10}{\frac{5}{\sqrt{10{,}000}}} = \frac{10}{\frac{1}{20}} = 200$

$MPL = \frac{5}{\sqrt{L}}$

$AC = \frac{TC}{q} = \frac{100(250) + 10(10{,}000)}{1000} = 125$

c) Now, assume that you can adjust your capital as well as labor. Calculate your optimal capital/labor choice.

$MPL = \frac{\sqrt{K}}{\sqrt{L}}$ $\quad MPK = \sqrt{\frac{L}{K}}$ $\quad 1000 = \sqrt{25KK}$

$= 5K$

$\frac{K}{L} = \frac{10}{250}$ $\quad 250K = 10L$ $\quad K = 200$

$L = 25K$ $\quad L = 5{,}000$