Problem Set 3
Econ 250

1

According to Bertrand's theory, price competition drives firms' profits down to zero even if there are only two competitors. Why don't we observe this in reality very often?

Product Differentiation, dynamic competition, and capacity constraints are the three most likely reasons.

2

Which model (Cournot, Bertrand) would you think provides a better approximation to each of the following industries: Oil refining, internet access, insurance. Why?

Capacity is a problem for oil companies, not so much for insurance. Given the logic of that, Cournot would fit oil better and insurance Bertand. Internet access has some speed (capacity constraints), but at the same time may compete in pricing against rivals. This day in age, internet is probably more toward Bertrand than Cournot.

3

In a duopoly with homogenous goods of golf balls, NEM competes with CEM, producing grosses of golf balls. The demand in the market for a gross is Q = 2400 - 2P. Once a firm has built capacity, it can produce up to its capacity with MC = 0. Building a unit of capacity costs 2000 (for NEM or CEM) and a unit of capacity lasts four years. The interest rate is 0 (the cost of capacity is spread out over all four years equally). Once production occurs each period the price in the market adjusts to the level at which all production is sold. (These firms compete on quantity, not price.)

a.) If NEM knew that CEM were going to build 200 units of capacity, how much would NEM want to build?
b.) How much profit does each firm make each period given that the cost of the capacity is spread out over all four years?

a.) $Q = q_1 + q_2$
$q_1 = 2400\text{-}2P - 200$ if q_2=200.
$P = 1100 - \frac{q_1}{2}$
$Pq_1 = 1100q_1 - q_1^2$
$MR = 1100 - q_1$
Setting MR = MC, $1100 - q_1 = 0$
q_1=1100, P=550

b.) π_1 = 550(1100) - 500*1100 = 55,000 per year
π_2 = 550(200) - 500*200 = 10,000 per year

4

Consider an industry where there are only two firms (a duopoly). The industry demand function is given by Q = 100 - $\frac{1}{3}$P (where P is price and Q is total quantity). Both firms have the following total cost function (where q denotes output): TC = 150 + 2q. Competition is Cournot style.

(a) Write down the profit function of each firm. (Hint: it will depend on the quantity the firm produces as well as the quantity the other firm produces)
(b) Calculate the reaction function of firm 1.
(c) What output should firm 1 produce if it expects its rival to produce 20 units?
(d) Find the Nash equilibrium.

a.) $\pi_1 = q_1$*P - (150 + 2q_1)
= q_1*(300-3q_1-3q_2) - 150 - 2q_1
= 298q_1 - 3q_1^2 - 3$q_2 q_1$ - 150
The symmetry of the firms implies the same with firm 2, except q_1 and q_2 are inverted.

b.) Firm 1 needs to maximize profit for any possible value of q_2
π_1 = 298q_1 - 3q_1^2 - 3$q_2 q_1$ - 150
Take a derivative wrt q_1.
0 = 298 - 6q_1 - 3q_2
$q_1 = \frac{149}{3} - \frac{1}{2}q_2$

c.) 119/3

d.) At a Nash equilibrium q_1=q_2 as the firms are identical. So:
$q_1 = \frac{149}{3} - \frac{1}{2}q_1$
$q_1 = q_2 = \frac{298}{9}$

5

In the town of Middleofnowhere there are only two farmers and they are the only producers of milk. The local demand for milk is given by (P denotes price measured in cents, Q denotes the total quantity measured in cartons): P = 2000 - 2Q. Both farmers have the same cost function given by (C is total cost measured in cents and q is output measured in cartons): C = 80,000 + 560 q.

(a) Calculate and draw the reaction (or best reply) function of firm 1 (that is, calculate the profit-maximizing output of firm 1 for every possible output of firm 2). Do the same for firm 2.

(b) Calculate the Cournot-Nash equilibrium (give the output of each firm, the total output, the price and the profit of each firm).

a.) P = 2000 - 2q_1 - 2q_2.
π_1=2000q_1 - 2q_1^2 - 2$q_1 q_2$-80,000-560q
First derivative of profit (same as MR=MC), 2000-4q_1-2q_2=560
$q_1 = \frac{1440-2q_2}{4}$
q_1=360-(1/2)q_2

As they are symmetric, the reverse applies to q_2.

b.)Because they are symmetric, q_1=q_2

(3/2)q_1=360

q_1=240, q_2=240, P=1040 $\pi_1 = \pi_2$=240*1040-80000-560*240=35,200.