

Don't Panic

Math 121C
Fall 2009
Instructor: Shawn Rafalski

Applied Calculus I
Exam 1

Write your name on this exam right now. Your work on this exam is to be your work alone. No calculators allowed. You have one hour to finish. Explain your answers clearly, and *show your work*. This exam has 7 pages, and the questions are worth a total of 100 points (not including bonus points). Only work on the bonus questions **after** you have tried to do all the regular questions. Don't forget to breathe regularly, and good luck!!

Begin working on the next page...

1. You run a business that produces top hats for penguins. Assume that the cost function $C(x)$ in dollars to produce x top hats is a linear function of x . Your business has \$400 in fixed costs, and the total cost to produce 5 top hats is \$520. You sell the hats for \$28 each at the South Pole.

(a) (10 points) Find the cost function $C(x)$.

$$520 = 5x + 400$$
$$24 = \frac{120}{5} = 24$$
$$\Rightarrow C(x) = 24x + 400$$

(b) (8 points) Find the revenue function $R(x)$. (Assume this function is linear.)

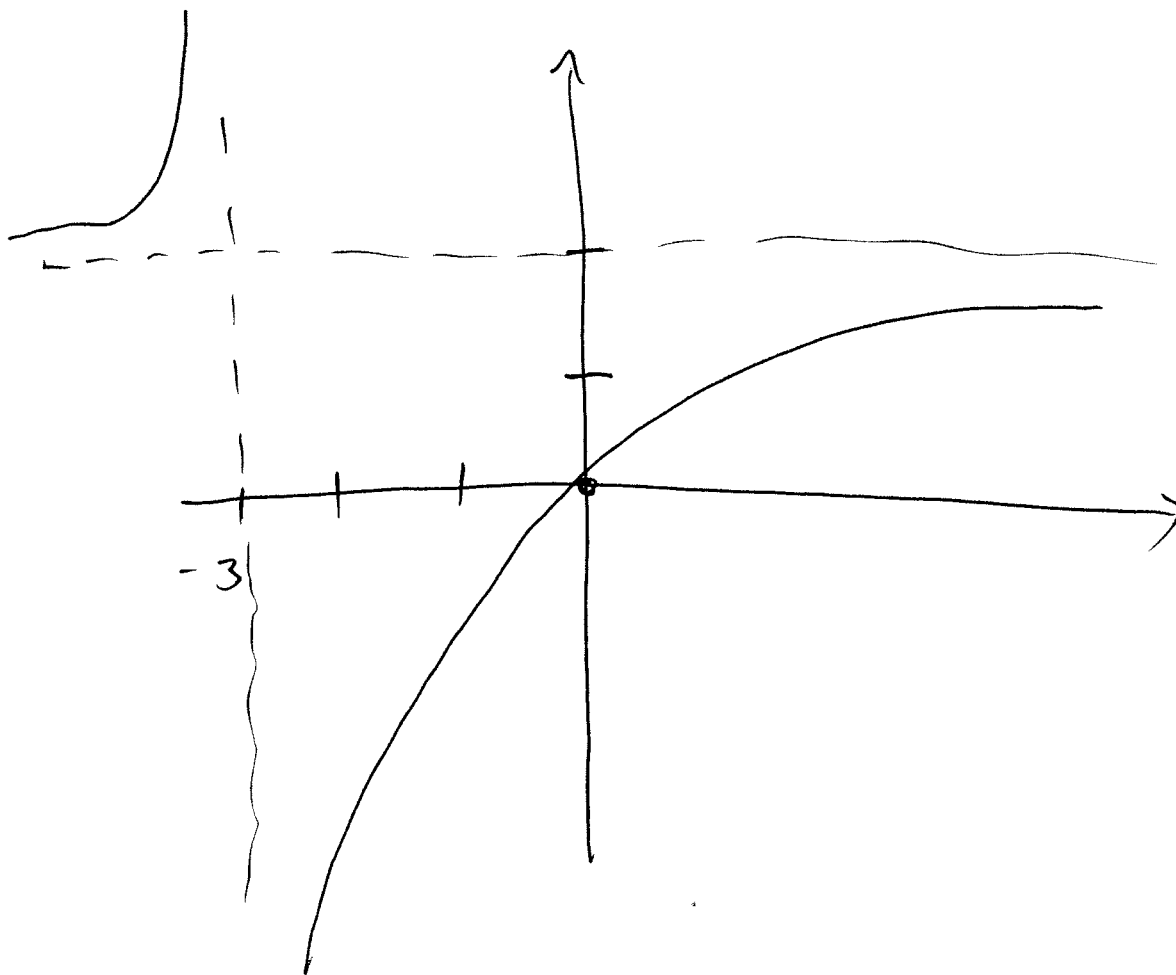
$$R(x) = 28x$$

(c) (12 points) Determine how many hats you have to manufacture in order to break even.

$$R = C \Rightarrow 28x = 24x + 400$$
$$4x = 400$$
$$x = 100 \text{ hats}$$

2. Let $f(x) = \frac{2x}{x+3}$.

- (a) (10 points) Draw the graph of this rational function, making sure to indicate the horizontal and vertical asymptotes, as well as the coordinates of at least one point on its graph.

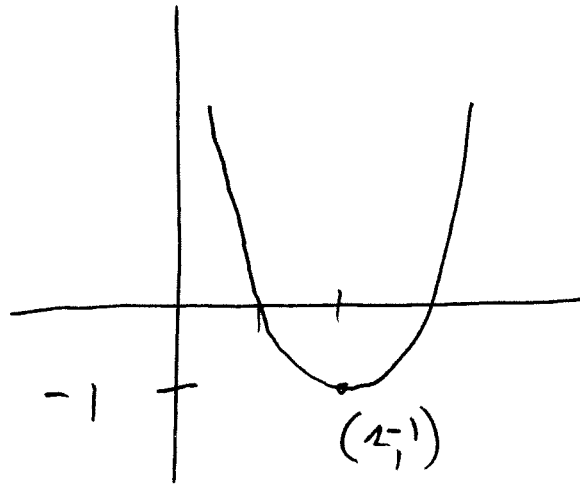


- (b) (5 points) What is the domain of f ?

$$x \neq -3$$

3. Let $p(x) = (x - 2)^2 - 1$.

(a) (10 points) Find the coordinates of the vertex of $p(x)$.



(b) (5 points) What is the range of $p(x)$?

$$[-1, \infty)$$

4. You have received a fat check for \$4000 in a lawsuit because someone insulted your dog. You decide to invest the money in an account paying 6.2% annual interest compounded continuously.

(a) (10 points) Write down the function $A(t)$ that gives the amount of money in the account after t years.

$$A(t) = 4000 e^{0.062t}$$

(b) (15 points) Determine the number of years it will take in order for the \$4000 to grow to \$10,000. (Aside from some basic cancelation, you should probably leave your answer to this question unsimplified).

$$10,000 = 4000 e^{0.062t}$$

$$\ln\left(\frac{10}{4}\right) = 0.062t$$

$$\frac{1}{0.062} \ln\left(\frac{10}{4}\right) = t$$

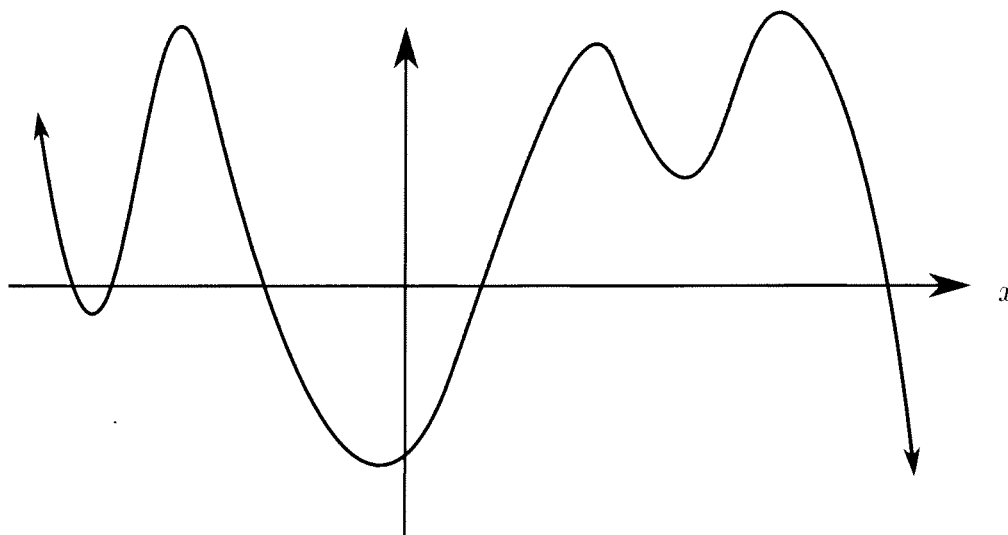
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years

- (c) **(Bonus: 5 points)** Answer part (b) again, but this time assume that your account compounds interest monthly.

$$10,000 = 4000 \left(1 + \frac{0.062}{12}\right)^{12t}$$

$$\ln\left(\frac{10}{4}\right) = 12t \ln\left(1 + \frac{0.062}{12}\right)$$

$$\frac{\ln\left(\frac{10}{4}\right)}{12 \ln\left(1 + \frac{0.062}{12}\right)} = t$$



5. The graph of a polynomial $p(x)$ is shown above.

- (a) (5 points) Determine whether the degree of p is odd or even, and explain your answer.

odd b/c the ends of the graph point in opposite directions

- (b) (5 points) Determine the smallest possible degree of p , and explain your answer.

turns + 1 = 6 + 1 = 7 is as small as the degree can be.

- (c) (5 points) Determine whether the leading coefficient of p is positive or negative, and explain your answer.

negative b/c the graph tends to $-\infty$ as x becomes large.