

Math 121C
Fall 2009
Instructor: Shawn Rafalski

Applied Calculus I
Exam 3

Write your name on this exam right now. Your work on this exam is to be your work alone. No calculators allowed. You have one hour to finish. Explain your answers clearly, and *show your work*. This exam has 6 pages, and the questions are worth a total of 100 points. Don't forget to breathe regularly, and good luck!!

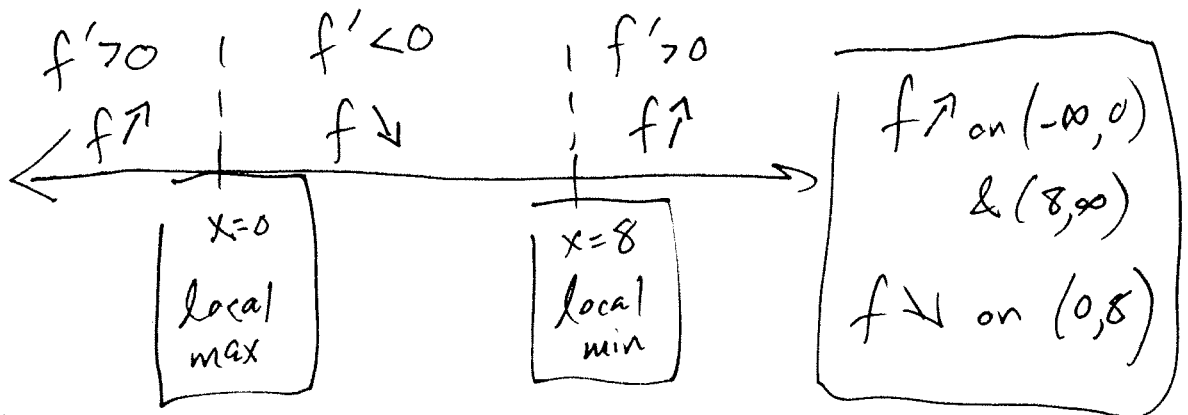
Begin working on the next page...

1. Consider the function $f(x) = \frac{x^3}{3} - 4x^2 + 11$.

(a) (15 points) Find the intervals on which f is increasing and decreasing, and find the x values of its local extrema.

$$f'(x) = x^2 - 8x$$

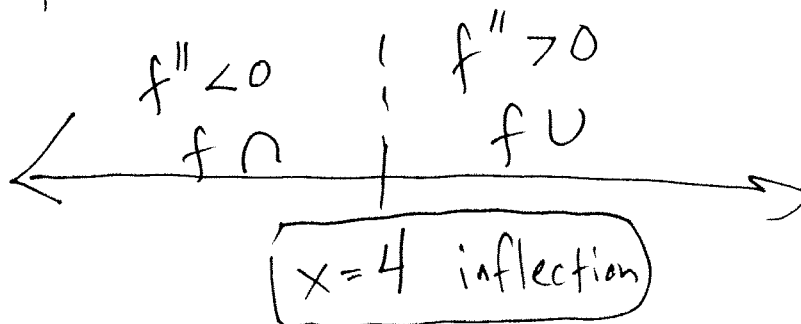
$$f'(x) = 0 : x^2 - 8x = 0 \rightarrow x(x-8) = 0 \quad \begin{array}{l} x=0 \\ x=8 \end{array}$$



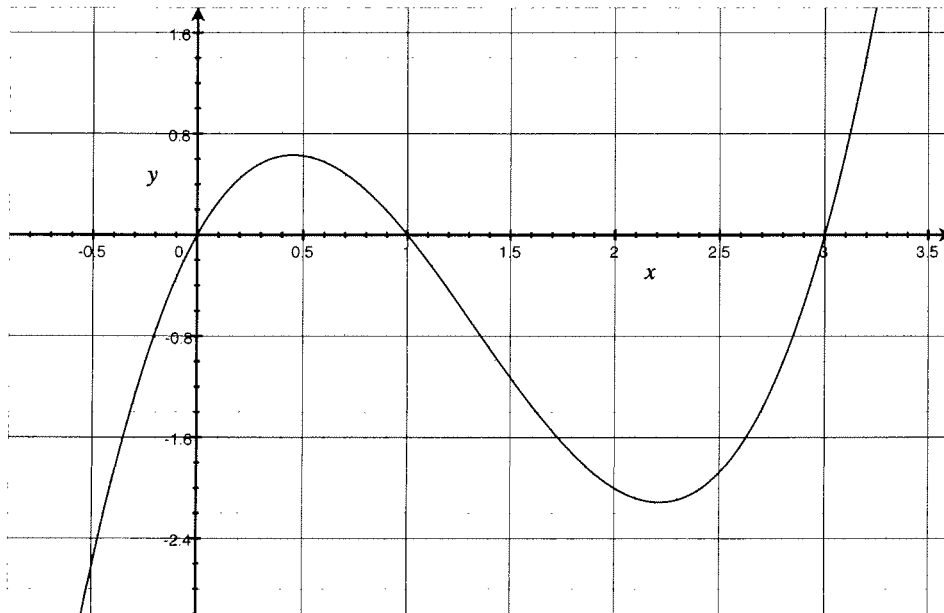
(b) (15 points) Find the intervals of concavity of f , and find any inflection points.

$$f''(x) = 2x - 8$$

$$f'' = 0 \text{ at } x = 4$$



f concave up on $(4, \infty)$
 f concave down on $(-\infty, 4)$



2. Use the graph above to answer the following questions. Your answers do not have to be exact. Just use your best estimates by looking at the graph.

(a) (10 points) If the graph above represents the function $f(x)$, then give the intervals on which f is increasing and decreasing.

$f \nearrow$ on $(-\infty, 0.45)$ & $(2.2, \infty)$

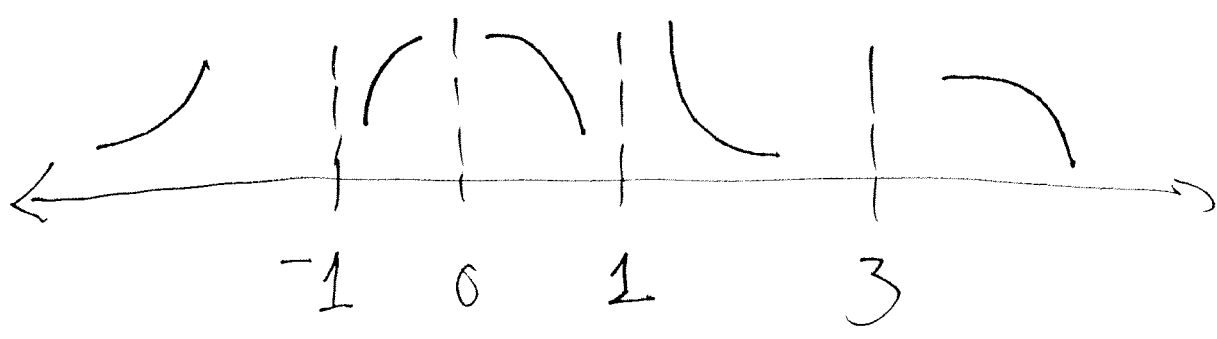
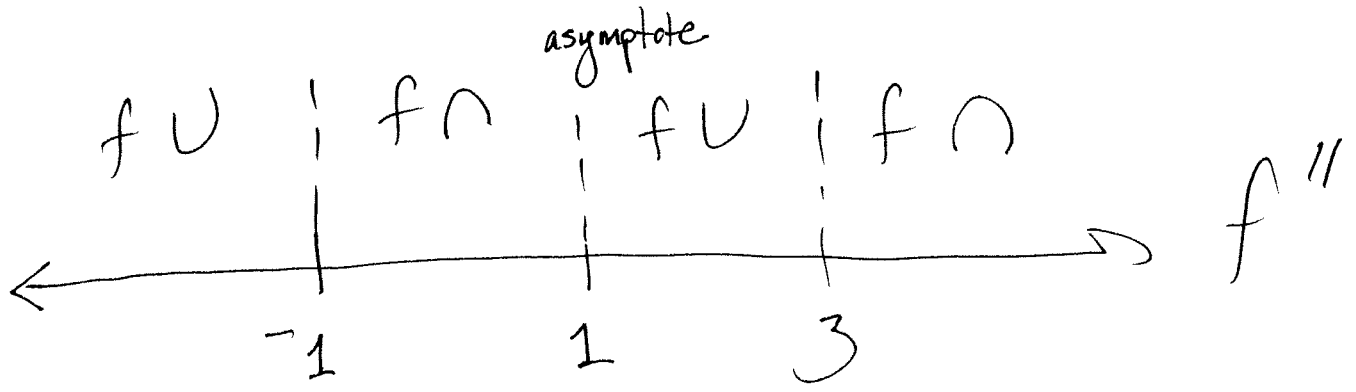
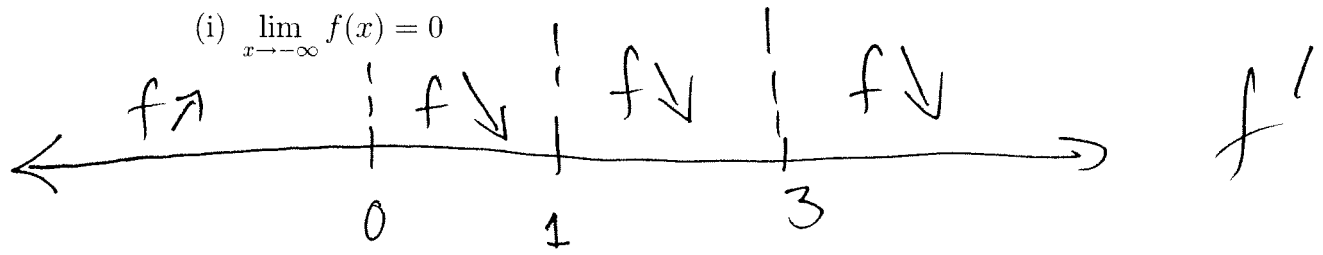
$f \searrow$ on $(0.45, 2.2)$.

(b) (10 points) If the graph above represents *the derivative* of the function $f(x)$, then give the intervals on which f is increasing and decreasing.

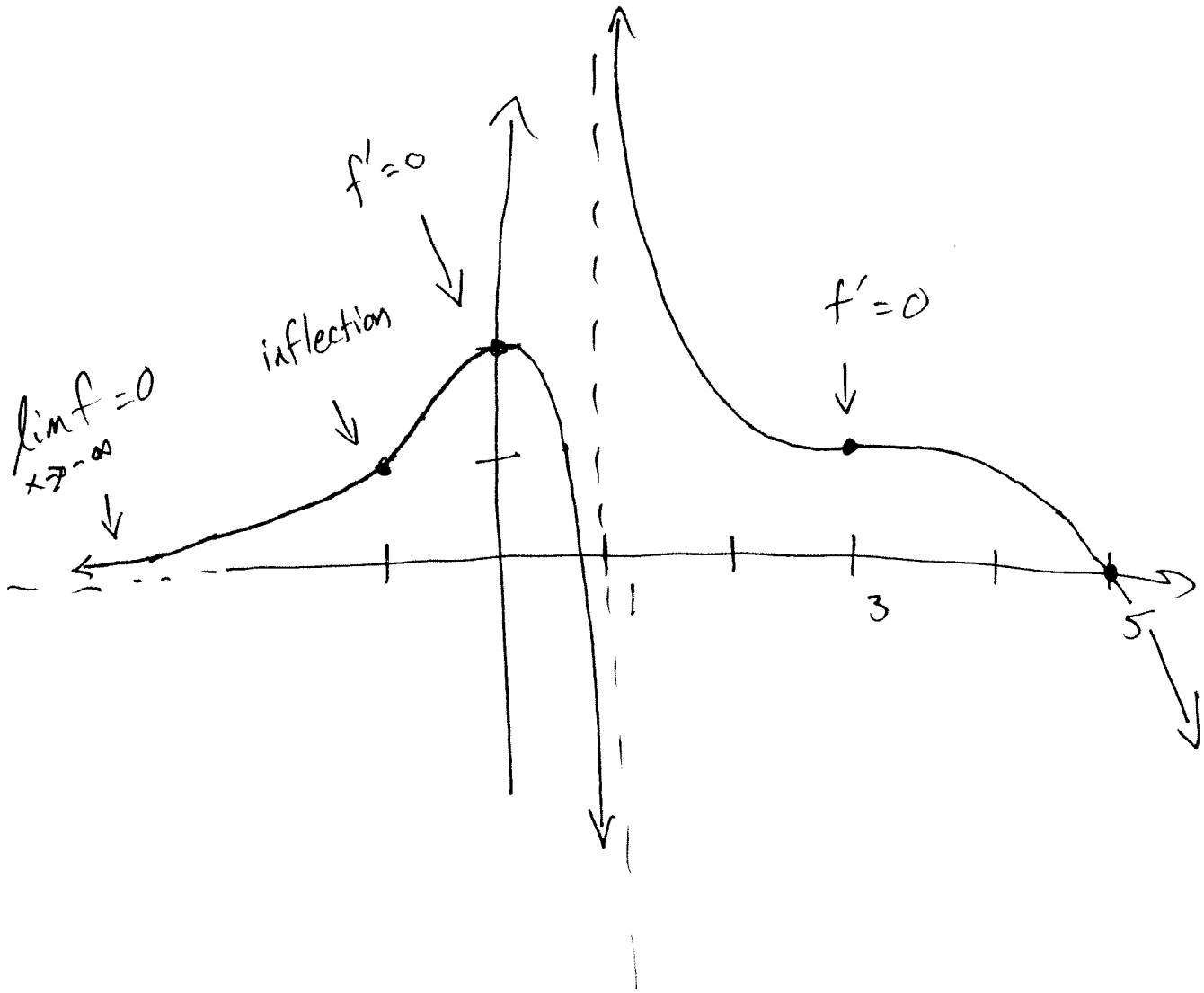
$f \nearrow$ if $f' > 0$ } $f \nearrow$ on $(0, 1)$ & $(3, \infty)$
 $f \searrow$ if $f' < 0$ } $f \searrow$ on $(-\infty, 0)$ & $(1, 3)$

3. (30 points) Sketch the graph of a single function $f(x)$ that satisfies all of the following properties. You may use the next page for additional work.

- (a) f is continuous and differentiable everywhere except at $x = 1$, where it has a vertical asymptote
- (b) $f(-1) = 1$, $f(0) = 2$, $f(3) = 1$, $f(5) = 0$
- (c) $f'(0) = 0$, $f'(3) = 0$
- (d) $f''(-1) = 0$, $f''(3) = 0$
- (e) $f'(x) > 0$ on $(-\infty, 0)$
- (f) $f'(x) < 0$ on $(0, 1)$, $(1, 3)$ and $(3, \infty)$
- (g) $f''(x) > 0$ on $(-\infty, -1)$ and $(1, 3)$
- (h) $f''(x) < 0$ on $(-1, 1)$ and $(3, \infty)$
- (i) $\lim_{x \rightarrow -\infty} f(x) = 0$



(Use this page for additional work.)



4. (20 points) Let x and y be two non-negative numbers that satisfy the relationship $400 = x + 2y$. Find the values of x and y that make the product xy as large as possible (this means, maximize the product), and determine the value of this largest possible product.

$$400 = x + 2y$$

Maximize $xy = P$ ← P stands for "product"

domain $0 \leq y \leq 200$

$$x = 400 - 2y \rightarrow P(y) = (400 - 2y)y$$

$$P'(y) = 400 - 4y$$
$$400y - 2y^2$$

$$P'(y) = 0 \rightarrow y = 100$$

$$x = 400 - 2y = 400 - 2 \cdot 100 = 200$$

$$x \cdot y = 100 \cdot 200 = 20,000$$

largest possible product