Math 121 C Fall 2009 HW 4

What got graded: §3.3 #30

§3.4 #20, 48

3.3 #30 $R(x) = 10x - 0.002x^2$ is the revenue (in 1000's of \$) for producing x units of an item.

(a) The average rate of change of revenue when production goes from 1000 to 1001 units:

$$\frac{R(1001) - R(1000)}{1001 - 1000} = \frac{8005.998 - 8000}{1} = 5.998$$

$$5.998 \left(\frac{1{,}000 \$}{\text{unit}} \right)$$

(b) Find the marginal revenue at $x = 1000$.

Solution:

$$\lim_{h\to 0} \frac{R(1000+h) - R(1000)}{h}$$

$$= \lim_{h\to 0} \frac{10(1000+h) - 0.002(1000+h)^2 - [10\cdot 1000 - 0.002(1000)^2]}{h}$$

$$= \lim_{h\to 0} \frac{10h - 0.002(2000)h - 0.002h^2}{h}$$

$$= \lim_{h\to 0} \left(10 - 0.002(2000) - 0.002h\right)$$

$$= 6 \left(\frac{1000\text{'s of }\$}{\text{unit}}\right)$$

(c) Find the additional revenue if production increases from 1000 to 1001 units:

$$R(1001) - R(1000) = 5.998 \ (1{,}000\text{'s of }\$)$$

(d) Compare (a) & (c) & (b).

(They are very similar.)

3.4 #20 (a) Find the equation of the secant line to the graph of $f(x) = 6 - x^2$ through the points $x = -1$, $x = 3$.

(b) Find the eq'n of the tangent line to $f(x) = 6 - x^2$ at $x = -1$.

<u>Soln:</u> (a) $\text{Slope} = \dfrac{f(3) - f(-1)}{3 - (-1)} = \dfrac{-3 - 5}{4} = -2$

So the line has equation

$$y - 5 = -2(x + 1)$$

(b) $\text{Slope} = \lim\limits_{h \to 0} \dfrac{f(-1+h) - f(-1)}{h} = 2$

~~Ar~~ So the equation of the tangent line is

$$\boxed{y-5=2(x+1)}$$

3.4 #48 $R(x)=20x-\frac{x^2}{500}$ is the revenue in \$ from selling x picnic tables.

(a) Find the marginal revenue when 1000 tables are sold:

$$R'(1000)=\lim_{h\to 0}\frac{R(1000+h)-R(1000)}{h}=16\ \text{\$/table}$$

(b) Estimate the revenue from the 1001st table sale using $R'(1000)$.

$R(1001)\approx \$16$.

(c) Determine the actual revenue from the sale of the 1001st table:

$$R(1001) - R(1000)$$

$$=$$

$$15.998 \ \$.$$

(d) Compare (b) and (c).

(They are very close).