Math 121 C Fall 2009

HW 8

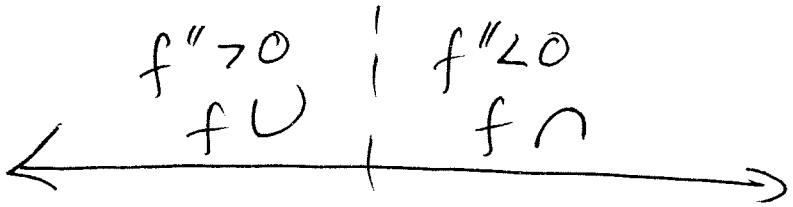
§5.3 #36, 42

§5.4 ~~#~~ 8

5.3 #36) $f(x) = -x^3 - 12x^2 - 45x + 2$

find the intervals of concavity and inflection points.

$f'' = -6x - 24$ $\quad f'' = 0$ at $x = -4$



-4

$f'' = 0$

So $f \cup$ on $(-\infty, -4)$

$f \cap$ on $(-4, \infty)$

$x = -4$ inflection pt

5.3 #42 $f = 2e^{-x^2}$

find intervals of concavity and inflection pts

Soln: $f'(x) = 2e^{-x^2} \cdot (-2x) = -4xe^{-x^2}$

$f''(x) = -4 \cdot e^{-x^2} + (-4x) \cdot e^{-x^2} \cdot (-2x)$

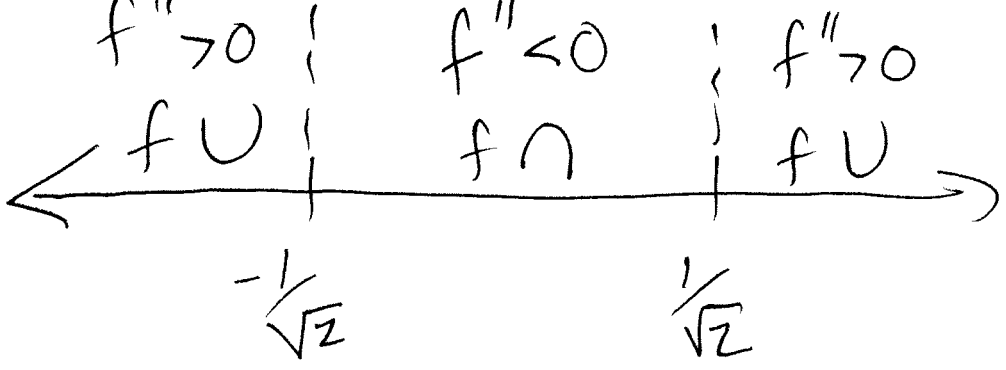
$f''(x) = -4e^{-x^2} + 8x^2 e^{-x^2}$

$f''(x) = 4e^{-x^2}\left[-1 + 2x^2\right]$

$f''(x) = 0$ if $-1 + 2x^2 = 0$

$\downarrow$

$x^2 = 1/2 \rightarrow x = \pm \frac{1}{\sqrt{2}}$



So f $\cup$ on $\left(-\infty, \frac{-1}{\sqrt{2}}\right)$ & $\left(\frac{1}{\sqrt{2}}, \infty\right)$

f $\cap$ on $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

and $x = \pm \frac{1}{\sqrt{2}}$ are inflection points.

5.4 #8 $f(x) = -x^4 + 6x^2$

Sketch.

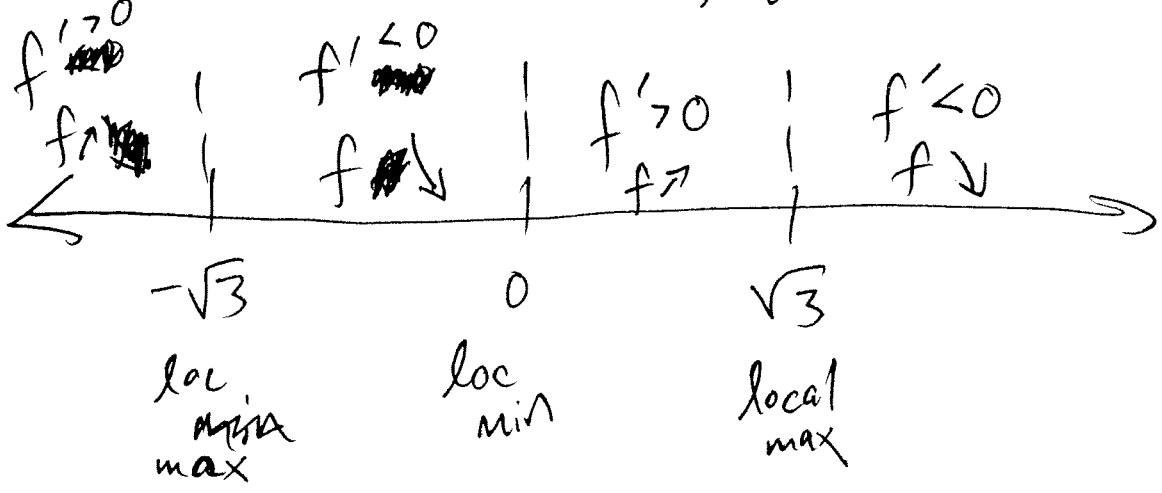
Solution: $f'(x) = -4x^3 + 12x$

$f''(x) = -12x^2 + 12$

$f' = 0 \rightarrow 0 = -4x^3 + 12x = -4x(x^2 - 3)$

$\downarrow$

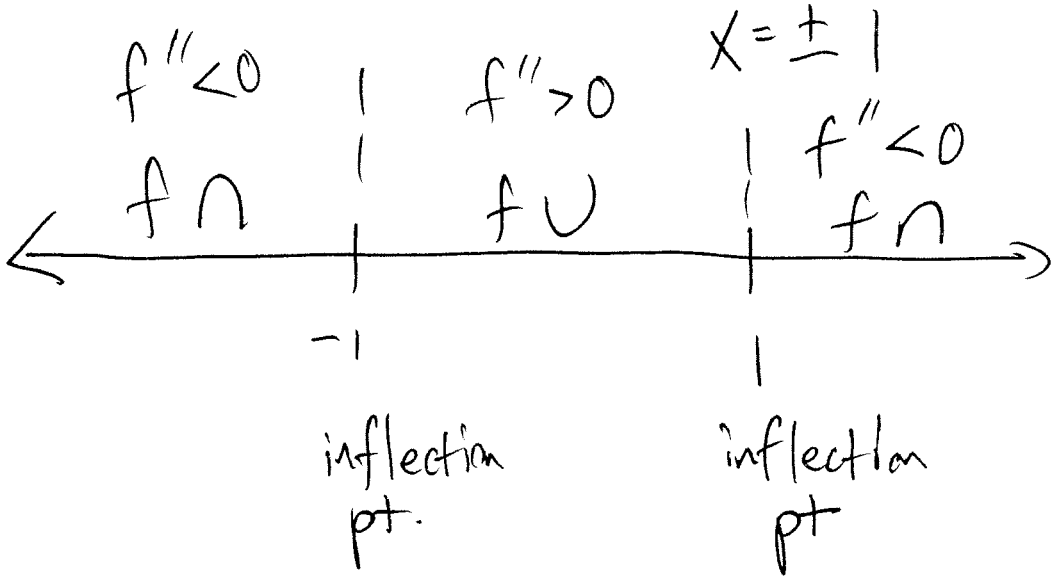
$x = 0$ or $x = \pm\sqrt{3}$



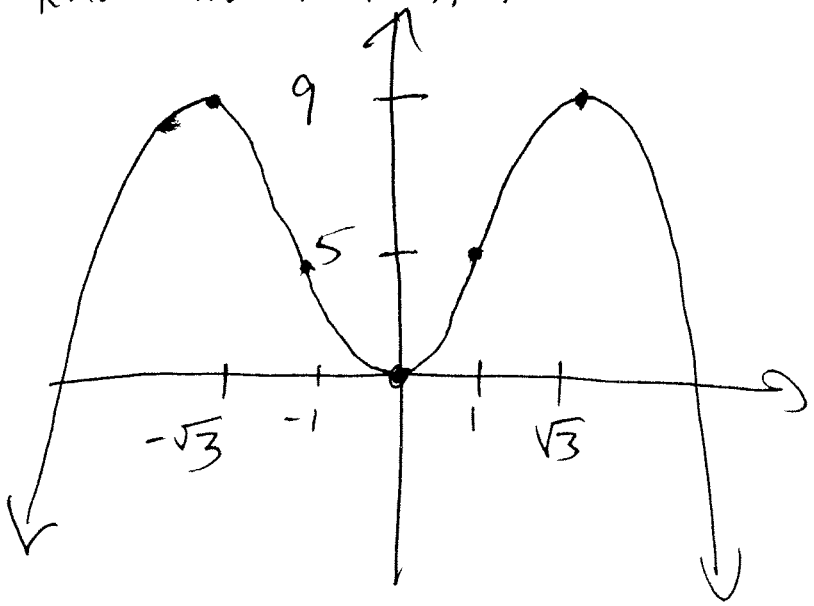
$f'' = 0 \rightarrow 0 = -12x^2 + 12 = -12(x^2 - 1)$

$\downarrow$

$x = \pm 1$



Plot all the zeros of f' & f'' and connect dots using what you know about f', f''.



$f(-\sqrt{3}) = -(-\sqrt{3})^4 + 6(-\sqrt{3})^2 = 9$

$f(\sqrt{3}) = 9$

$f(0) = 0$

$f(\pm 1) = 5$