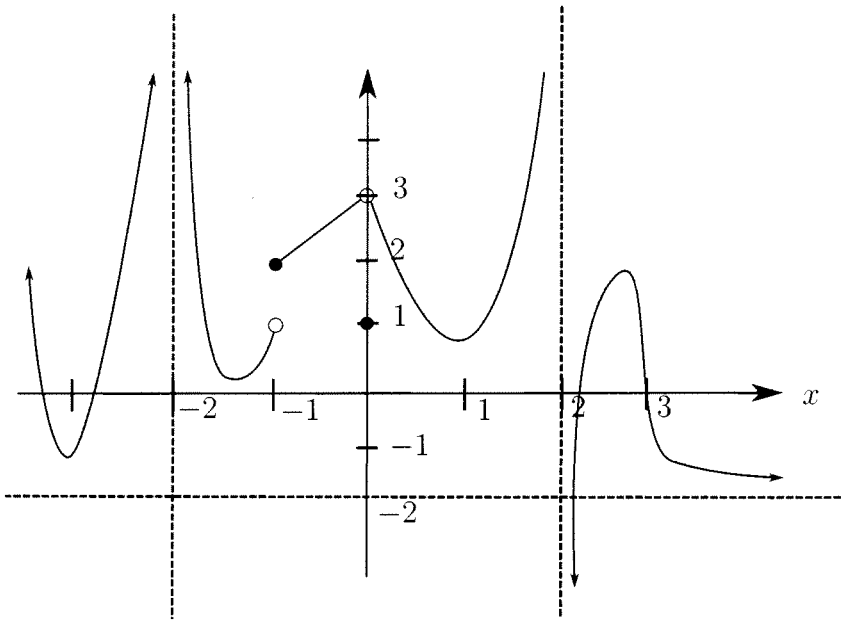
Don't Panic

Math 171A
Fall 2009
Instructor: Shawn Rafalski

Differential Calculus
Exam 1

Write your name on this exam right now. Your work on this exam is to be your work alone. No calculators allowed. You have one hour to finish. Explain your answers clearly, and *show your work.* This exam has 7 pages, and the questions are worth a total of 100 points (not including bonus points). Only work on the bonus questions **after** you have tried to do all the regular questions. Don't forget to breathe regularly, and good luck!!

Begin working on the next page.



1. (4 points each) The above figure shows the graph of the function $f(x)$. Determine the following:

   (a) $\lim_{x \to 2} f(x)$ DNE.

   (b) $\lim_{x \to -1^+} f(x) = 2$

   (c) $\lim_{x \to 3} f(x) = 0$

   (d) $\lim_{x \to -2} f(x) = \infty$

   (e) $f(0) = 1$

(do the easy questions first...)

2. (a) (15 points) State the ϵ-δ definition of what it means for a function $f(x)$ to have the limit L as x approaches the value a. If you cannot remember the exact definition, try to say in your own words what the definition says.

$\lim_{x \to a} f = L$ if for every $\epsilon > 0$ there is a $\delta > 0$ so that

if $0 < |x-a| < \delta$ then $|f(x) - L| < \epsilon$

(b) (15 points) Use the ϵ-δ definition of the limit to prove that $\lim_{x \to 2}(3x - 1) = 5$.

(1) Want $|f(x) - 5| < \epsilon$

$\Downarrow$

$|3x - 1 - 5| < \epsilon$

$\Downarrow$

$|3x - 6| < \epsilon$

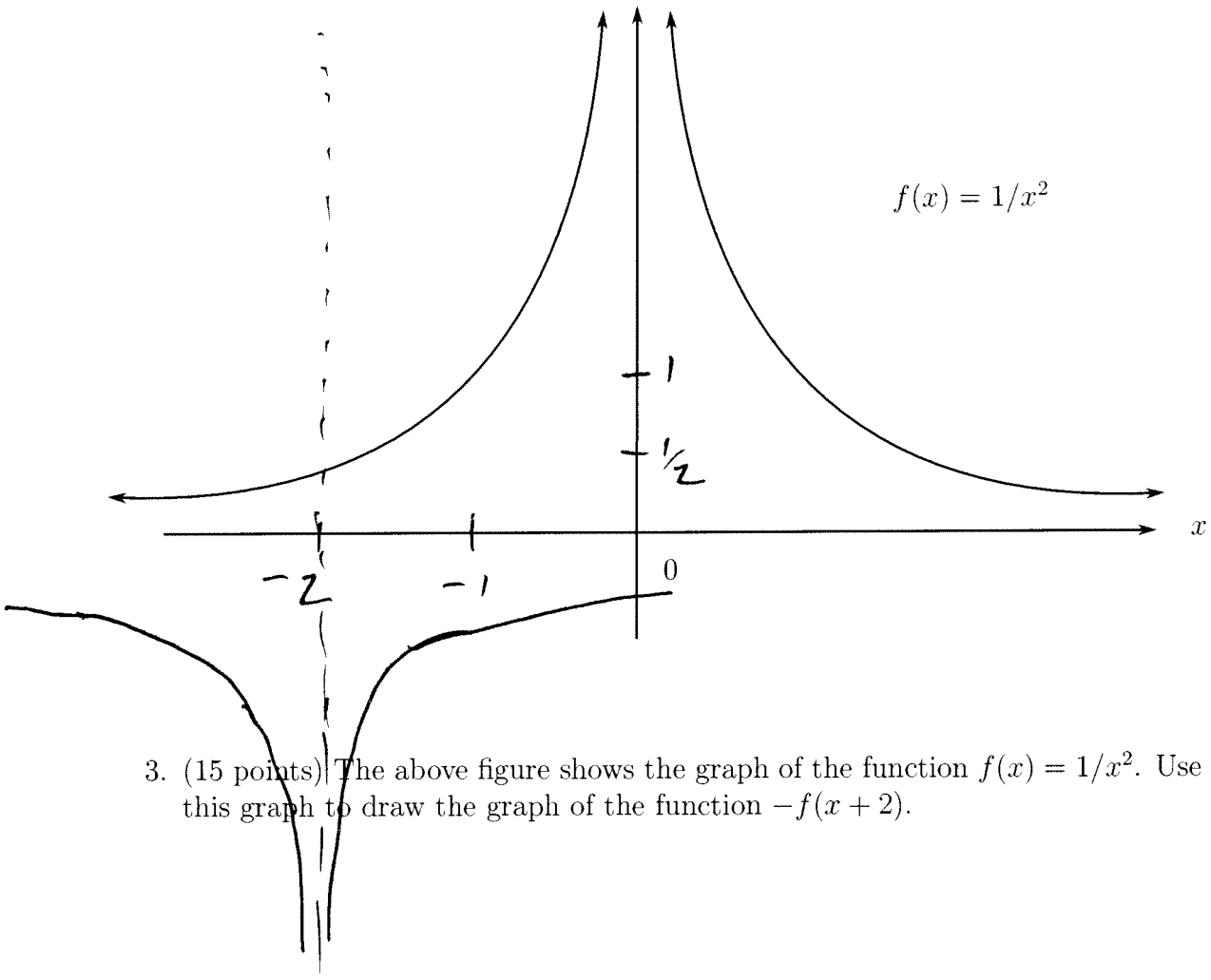
$\Downarrow$

$3|x-2| < \epsilon \rightarrow |x-2| < \epsilon/3$

(2) So choose $\delta = \epsilon/3$.

If $|x-2| < \delta = \epsilon/3$ then $3|x-2| < \epsilon$

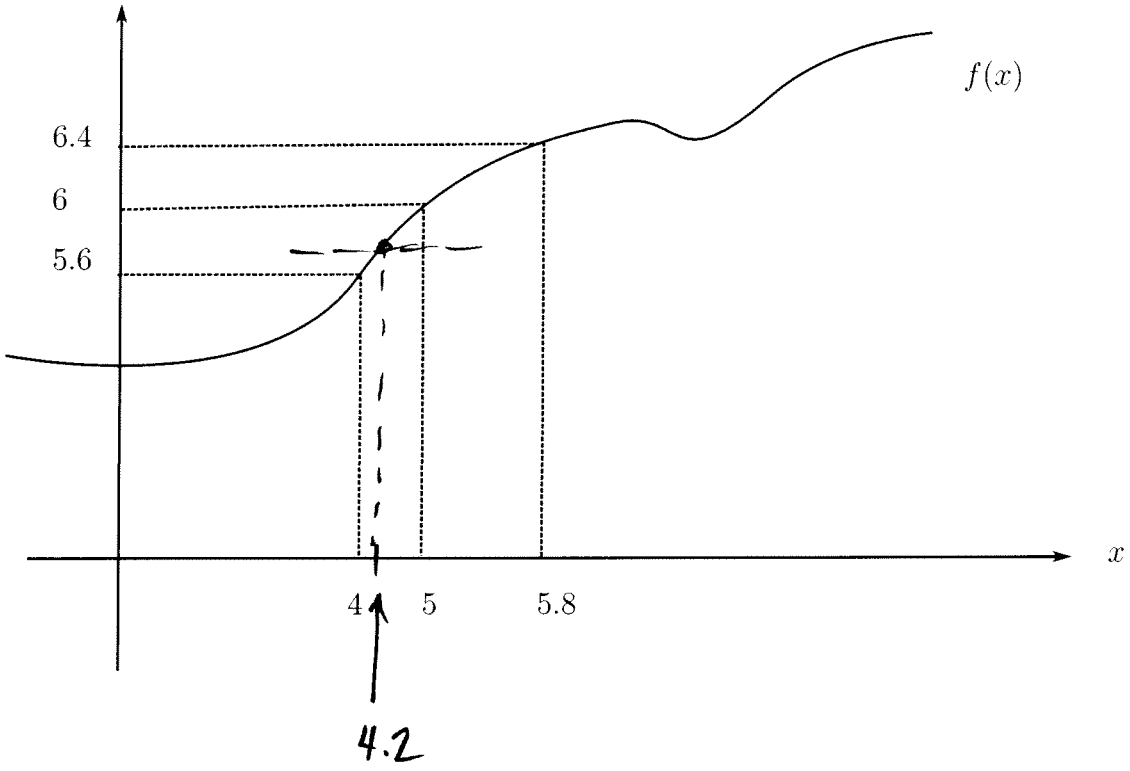
and $3|x-2| = |f(x)-5|$. Done.



3. (15 points) The above figure shows the graph of the function $f(x) = 1/x^2$. Use this graph to draw the graph of the function $-f(x+2)$.

$f(x+2)$ is $f(x)$ shifted 2 units left.

$-f(x+2)$ reflects this along the x-axis.



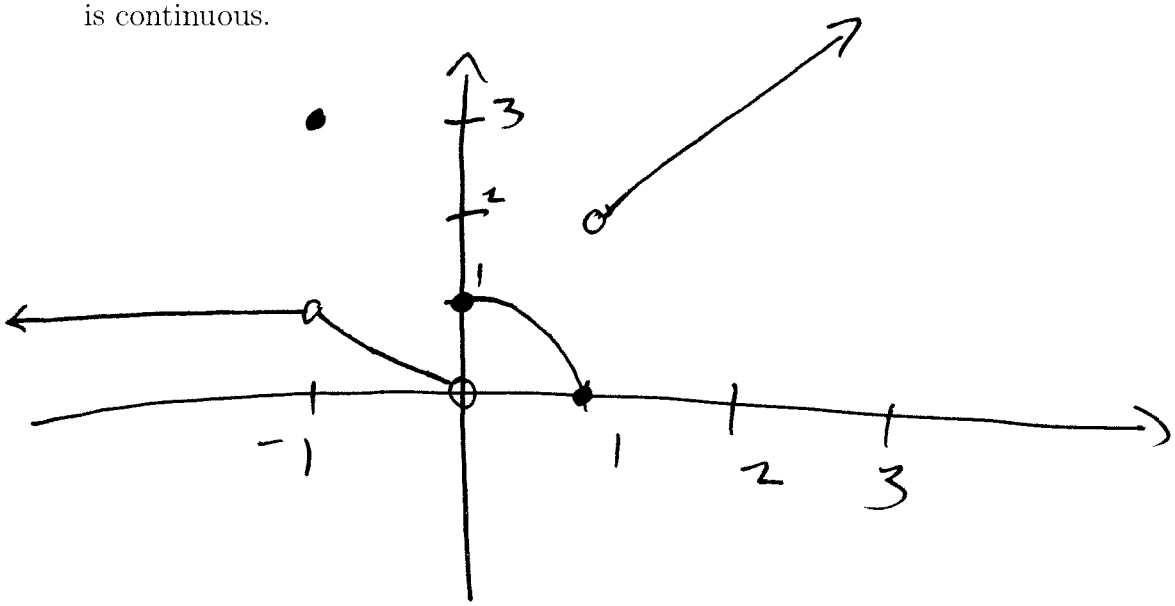
4. (10 points) Use the graph of the function $f(x)$ above in order to find a positive number δ so that if $|x-5|<\delta$, then $|f(x)-6|<0.4$.

If $\delta = .8$ then $4.2 < x < 5.8$ and then $f(x)$ will be within 0.4 of 6.

5. (25 points) Let T be the piecewise-defined function

$$T(x) = \begin{cases} 1 & \text{if } x < -1 \\ 3 & \text{if } x = -1 \\ -x & \text{if } -1 < x < 0 \\ 1 - x^2 & \text{if } 0 \leq x \leq 1 \\ x + 1 & \text{if } 1 < x \end{cases}$$

Graph $T(x)$, and use the graph to determine all the points where the function is continuous.



So T is cts on

$(-\infty, -1) \cup (-1, 0) \cup [0, 1] \cup (1, \infty)$

6. (**Bonus 8 points!**) Prove that $\lim_{x \to 0} \frac{1}{x^2} = \infty$, using the definition of an infinite limit. (The graph of this function is pictured on page 4). Partial bonus points for drawing the correct picuture, even if you can't remember the exact definition of an infinite limit.