M 171 A Fall 2009 HW 3

What got graded: § 2.2 # 12

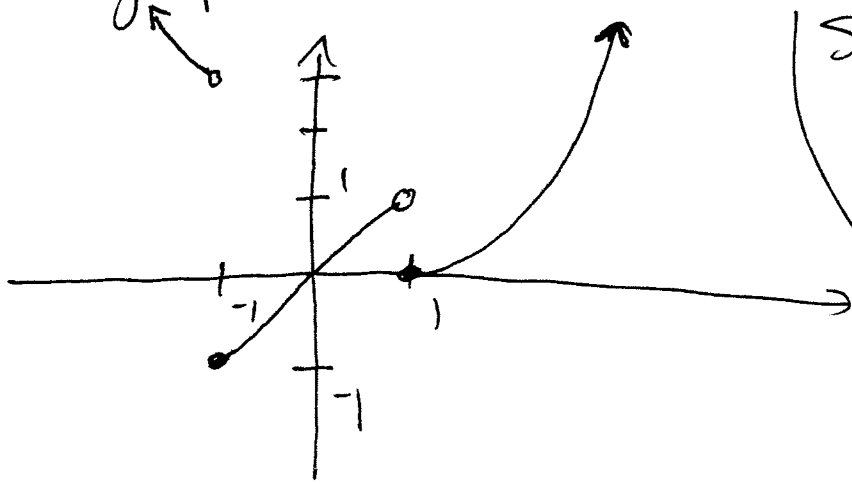
§ 2.3 # 20, 36

#12 Sketch the graph of

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ (x-1)^2 & \text{if } x \ge 1 \end{cases}$$

and use it to find the values a for which $\lim_{x \to a} f$ exists.

Soln: The graph of f is



So $\lim_{x \to a} f$ dne at $a = -1, +1$

#20 $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$

$$= \lim_{h\to 0} \frac{8+12h+6h^2+h^3-8}{h}$$

$$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$$

$$= \lim_{h\to 0} 12+6h+h^2$$

$$= 12+6\cdot 0+0^2 = \boxed{12}$$

(36) If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all $x \geq 0$, then what is

$$\lim_{x \to 1} g \,?$$

Sol'n: Since $2x$ & $x^4 - x^2 + 2$ are polynomials, it's true that

$$\lim_{x \to 1} 2x = 2 \cdot 1 = 2$$

&

$$\lim_{x \to 1} x^4 - x^2 + 2 = 1^4 - 1^2 + 2 = 2$$

Singe $g(x)$ is squee~~zed~~d between these functions, $\boxed{\lim_{x \to 1} g(x) \text{ is also } 2}$, by the Squeeze Theorem.