HW 6 Sol'n Math 171

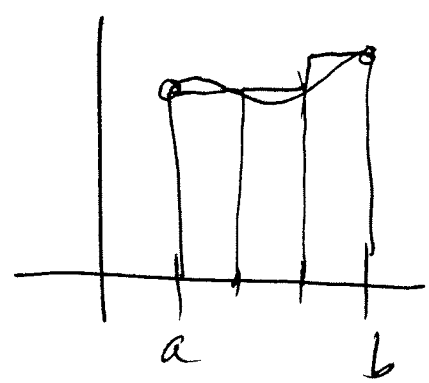
5.1 #20 Determine the region whose area equals the given limit.

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n} \frac{2}{n}\left(5+i\cdot\frac{2}{n}\right)^{10}\right) = \underbrace{\int_5^7 x^{10}\,dx}_{\text{Answer.}}$$

Work backwards:

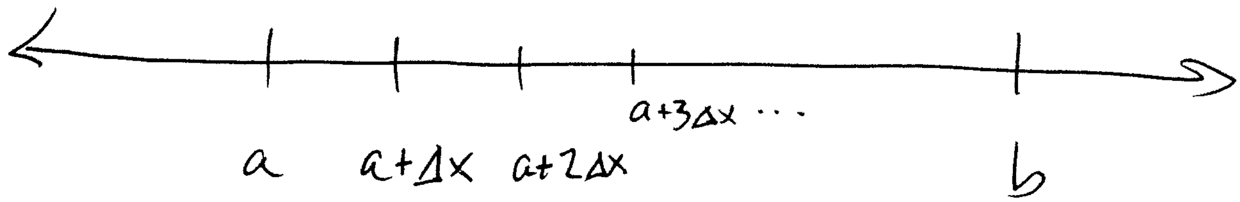
A limit of Riemann sums looks like

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n} f(x_i^*)\,\Delta x\right)$$



where $\Delta x = \frac{b-a}{n}$

and x_i^* is in $[x_{i-1}, x_i]$



In general $\quad x_{i-1} = a + (i-1)\Delta x$

$$x_i = a + i\Delta x$$

and $x_i^* = a + i\Delta x$ $\underline{\underline{\text{if}}}$ we are using the right-hand endpoint of $[x_{i-1}, x_i]$

So $\dfrac{b-a}{n} = \dfrac{2}{n}$ means $b-a=2$.

And $5 + i \cdot \frac{2}{n}$ has the form

$a + i\Delta x$ with $a=5$ and $\Delta x = \frac{2}{n}$.

So $b=7$ (b/c $b-a=2$) and

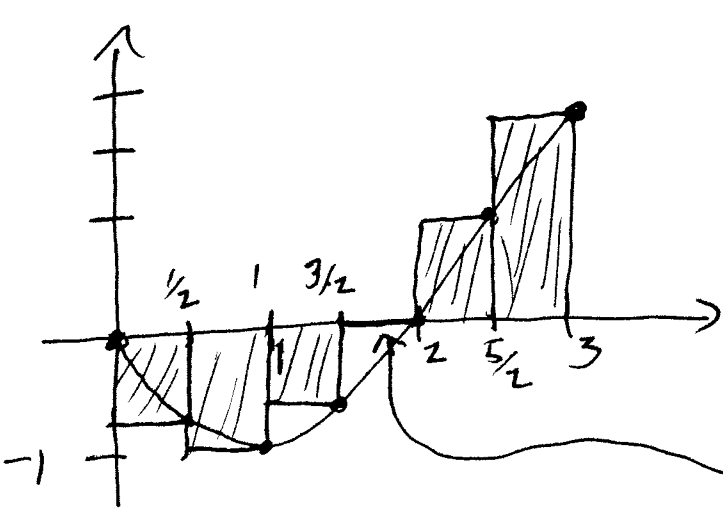
$f(x) = x^{10}$ b/c we see

$(5 + i\Delta x)^{10}$ in the sum.

So the limit equals $\boxed{\int_5^7 x^{10}\,dx}$.

5.2 #2 $f(x) = x^2 - 2x$, $0 \le x \le 3$

evaluate R_6 the right-hand Riemann sum with $n=6$.



The area is the sum of these rectangles, one of which is height zero.

$\Delta x = 1/2$. ~~So~~ And

$x_i^* =$ right-hand endpt

$x_1^* = 1/2 \qquad x_3^* = 3/2 \qquad x_5^* = 5/2$

$x_2^* = 1 \qquad x_4^* = 2 \qquad x_6^* = 3$

$$R_6 = \frac{1}{2} \cdot \Big[f(1/2) + f(1) + f(3/2) + f(2) + f(5/2) + f(3) \Big]$$

$$= \frac{1}{2}\Big[\frac{1}{4} - 1 + 1 - 2 + \frac{9}{4} - 3 + 0 + \frac{25}{4} - 5 + 9 - 6 \Big]$$

$$= \frac{1}{2}\Big[\frac{35}{4} - 7 \Big] = \frac{1}{2}\Big[\frac{35}{4} - \frac{28}{4} \Big] = \boxed{\frac{7}{8}}$$

5.2 #21 Evaluate $\int_{-1}^{5} (1+3x)\,dx$

using the form of the definition from the text.

$$\frac{b-a}{n} = \frac{5-(-1)}{n} = 6/n$$

$$x_i^* = -1 + i\,\frac{6}{n}$$

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n} \frac{6}{n}\left[1+3\left(-1+i\frac{6}{n}\right)\right]\right)$$

$$= \lim_{n\to\infty}\left(\sum_{i=1}^{n} \frac{6}{n}\left(-2+\frac{18i}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(-\frac{12}{n}\cdot\sum_{i=1}^{n} 1 + \frac{18}{n}\cdot\frac{6}{n}\sum_{i=1}^{n} i\right)$$

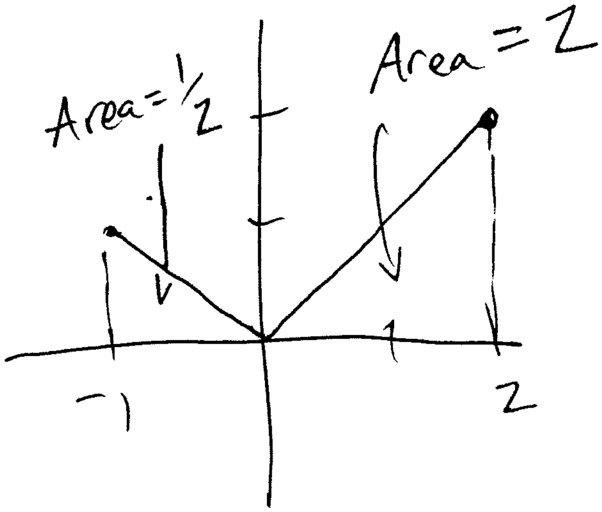
$$= \lim_{n\to\infty} \left(\frac{-12}{n} \cdot n + \frac{18 \cdot 6}{n^2} \cdot \frac{n(n+1)}{2} \right)$$

$$= \lim_{n\to\infty} \left(-12 + \frac{54\, n(n+1)}{n^2} \right)$$

$$= -12 + 54 = \boxed{42}$$

5.2 #39

$$\int_{-1}^{2} |x|\, dx$$ evaluate using areas



$$\int_{-1}^{2} |x|\, dx = 5/2.$$

5.3 #18

$$y = \int_{1/x^2}^{0} \sin^3 t \, dt$$

$$\frac{dy}{dx} = ? \quad : \quad y = -\int_{0}^{1/x^2} \sin^3 t \, dt$$

$$\frac{dy}{dx} = -\frac{d}{dx}\left[\int_{0}^{1/x^2} \sin^3 t \, dt\right]$$

$$= -\sin^3\left(\frac{1}{x^2}\right) \cdot \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \boxed{-\sin^3\left(\frac{1}{x^2}\right) \cdot \left(\frac{-2}{x^3}\right)}$$