M171A Fall 2009 Quiz 3

① Use the defn of the derivative to find the derivative of $f(x) = x^2 + 1$.

Soln:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} (2x + h) = 2x.$$

So $f'(x) = 2x$.

② Find $f'(x)$ if $f(x) = \dfrac{\frac{2}{x^3} + \sqrt{x^3}}{x^2+1}$.

Sol'n: $f(x) = \dfrac{2x^{-3} + x^{3/2}}{x^2+1}$, so use quotient rule:

$$f'(x) = \frac{(-6x^{-4} + \frac{3}{2}x^{1/2})(x^2+1) - (2x^{-3} + x^{3/2})\cdot 2x}{(x^2+1)^2}.$$