

$$f(x) = \frac{2x}{x+4}$$

1. Answer the following question: What does the Mean Value Theorem say about the function $\frac{2x}{x+4}$ on the closed interval $[0, 4]$? Also, find all the numbers c that satisfy the conclusion of the Mean Value Theorem.

MVT says there is at least one point $0 < c < 4$ where $f'(c) = \frac{f(4) - f(0)}{4 - 0} = \frac{1}{4}$.

$$f'(x) = \frac{2(x+4) - 2x}{(x+4)^2} = \frac{8}{(x+4)^2} \quad \frac{8}{(x+4)^2} = \frac{1}{4}$$

$$32 = (x+4)^2 = x^2 + 8x + 16 \rightarrow x^2 + 8x - 16 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 4 \cdot (-16)}}{2} = \frac{-8 \pm \sqrt{128}}{2} \rightarrow \frac{-8 + \sqrt{128}}{2} = \boxed{-4 + 4\sqrt{2}}$$

$\rightarrow -8 - \frac{\sqrt{128}}{2}$ not in $[0, 4]$

2. Draw the graph of a function that has a vertical asymptote at $x = 2$, passes through the points $(0, 0)$ and $(3, 4)$, is concave up and increasing on $(-\infty, 2)$, and concave up and decreasing on $(2, \infty)$.

