M 271 A Fall 2009 HW 2 Solutions

What got graded: § 11.2 # 9, 14

§ 11.3 # 16, 22

(9) Find an equation of the tangent(s) to the curve $x = 6\sin t$, $y = t^2 + t$ at $(0,0)$ and graph the curve and tangents.

Sol'n: At $(0,0)$, $t = 0$.

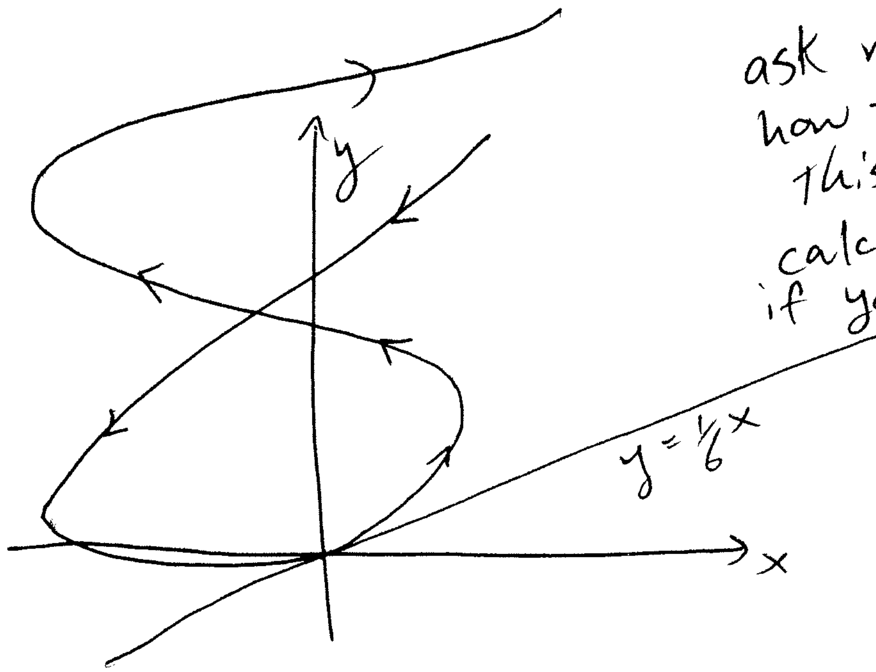
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+1}{6\cos t}.$$

At $t=0$, $dy/dx = 1/6$.

So the tangent equation is

$$y - 0 = \tfrac{1}{6}(x - 0) \rightsquigarrow \boxed{y = \tfrac{1}{6}x}$$

Using a calculator, the graph looks like



ask me how to draw this on your calculator if you can't figure it out :)

(14) $x = t + \ln t \qquad y = t - \ln t$.

Find dy/dx & d^2y/dx^2 and the values of t for which the curve is concave up.

Sol'n: $dy/dx = \dfrac{1 - 1/t}{1 + 1/t} = \dfrac{t-1}{t+1}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{t-1}{t+1}\right)}{dx/dt} = \frac{\left[(t+1) - (t-1)\right]/(t+1)^2}{1 + \frac{1}{t}}$$

$$= \frac{2}{(t+1)^2(1+\frac{1}{t})} = \frac{2t}{(t+1)^3}$$

When is $\frac{2t}{(t+1)^3} > 0$?

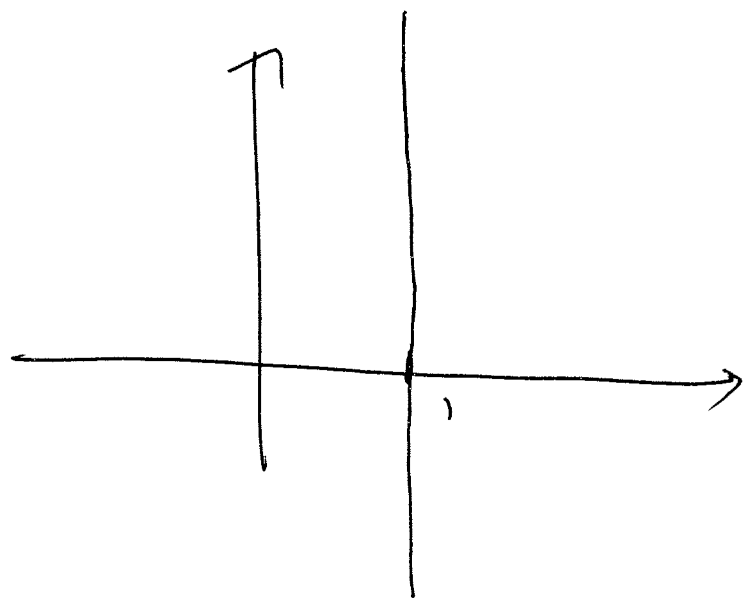
when t & $t+1$ are positive, so $t>0$.

<u>or</u> when t & $t+1$ are negative, but $\ln t$ does not take negative #'s so

$(0, \infty)$ ← is the interval where the curve is concave up

(16) Identify the curve $r\cos\theta = 1$.

Sol'n: Since $x = r\cos\theta$, this curve is $x = 1$.



(22) Find a polar equation for $x^2+y^2=9$.

Sol'n: $x^2+y^2=r^2$ so $r^2=9$ or $r=\pm 3$.

This is a circle: