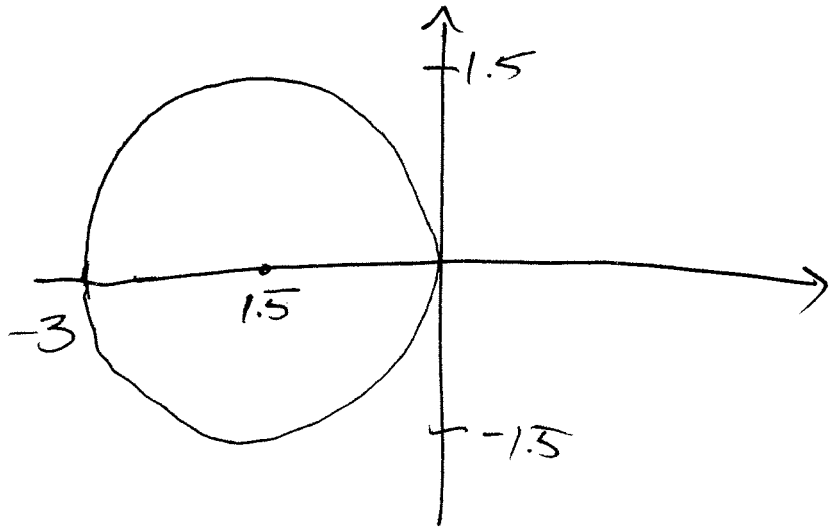
Math 271A Fall 2009 HW 3

What got graded: §11.3 # ~~6~~ 32, 50

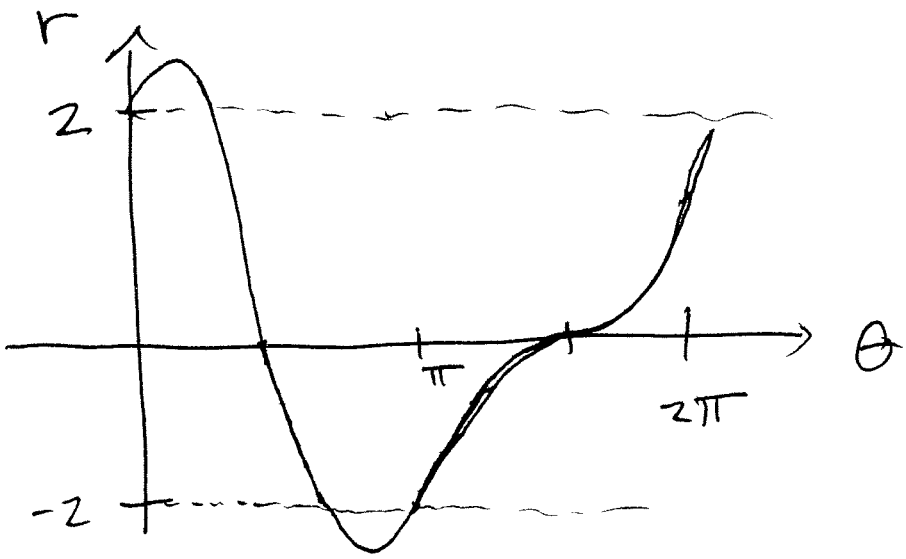
§11.4 # ~~32, 50~~ 6

(#~~6~~32) Sketch $r = -3\cos\theta$.

We know $r = 3\cos\theta$ is a circle with radius $\frac{3}{2}$ centered at the point with rectangular coordinates $(\frac{3}{2}, 0)$. Changing the r-coordinate by a minus sign just moves the circle:



(50) Sketch the polar curve for the graph $r = f(\theta)$ shown here



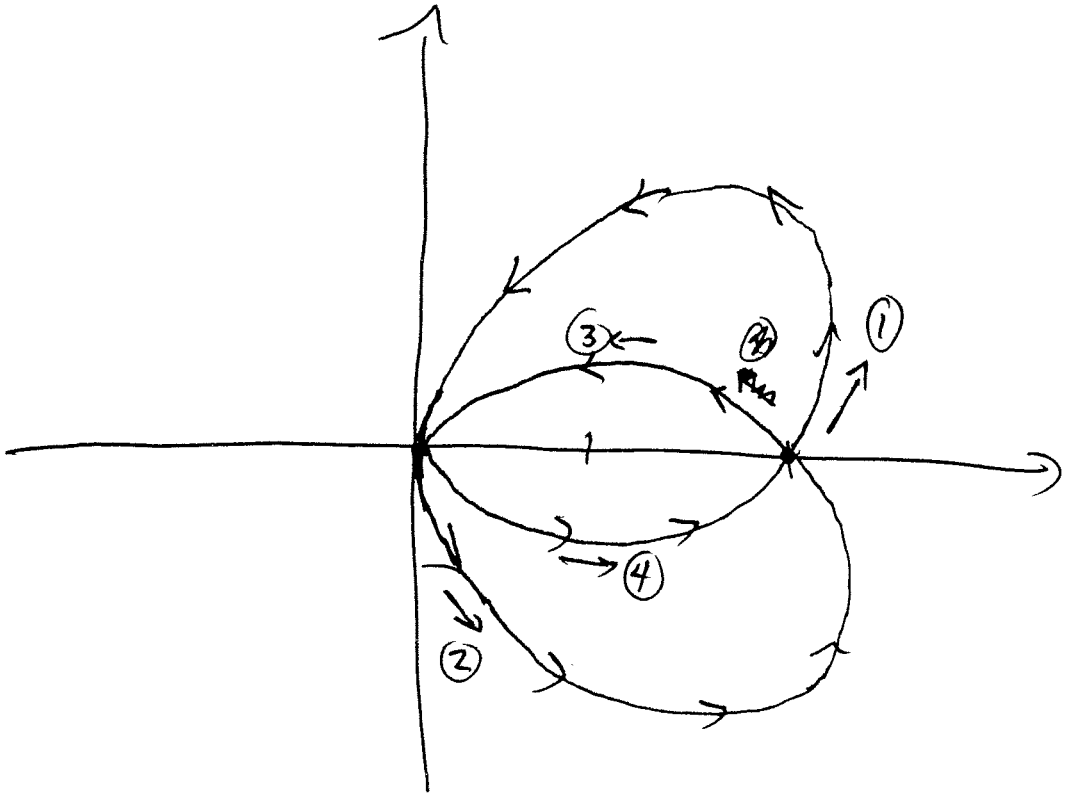
Soln: We use the symmetries of this graph to draw the polar curve. Notice that is looks like $r(\theta)$ ~~looks like~~ for $\pi/2 \le \theta \le \pi$ or $\frac{3\pi}{2} \le \theta \le 2\pi$

is $-r(\theta)$ for $0 \le \theta \le \pi/2$.

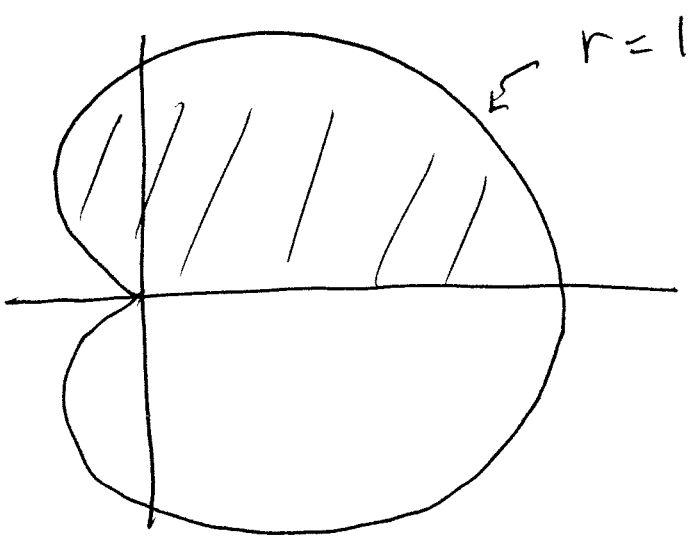
and

$\pi \le \theta \le \frac{3\pi}{2}$.

So the graph of this polar curve should have these symmetries:



~~#6~~ Find the area of



Sol'n: The shaded region occurs from $\theta = 0$ to $\theta = \pi$, so the area is

$$\int_0^{\pi} \frac{(1+\cos\theta)^2}{2}\, d\theta$$

$$= \int_0^{\pi} \frac{1 + 2\cos\theta + \cos^2\theta}{2}\, d\theta$$

$$= \frac{\pi}{2} + \int_0^{\pi} \cos\theta\, d\theta + \frac{1}{2}\int_0^{\pi} \cos^2\theta\, d\theta$$

$$= \pi/2 + \sin\theta \Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi} \left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] d\theta$$

$$= \frac{\pi}{2} + \frac{\pi}{4} + \left(\frac{\sin 2\theta}{4}\right)\Big|_0^{\pi} = \boxed{\frac{3\pi}{4}}$$