M271 A Fall 2009 Hw 4

<u>Graded:</u> §13.5 #10, 30, 62

(10) Find parametric/symmetric equations for the line through $(2,1,0)$ and perpendicular to both $\vec{i}+\vec{j}$ and $\vec{j}+\vec{k}$.

<u>Soln:</u> point $= (2,1,0)$. vector $\vec{v} = (\vec{i}+\vec{j}) \times (\vec{j}+\vec{k})$

$$= \vec{i} \times (\vec{j}+\vec{k}) + \vec{j} \times (\vec{j}+\vec{k})$$

$$= \vec{i}\times\vec{j} + \vec{i}\times\vec{k} + \vec{j}\times\vec{j} + \vec{j}\times\vec{k}$$

$$= \vec{k} - \vec{j} + \vec{0} + \vec{i} = \vec{i} - \vec{j} + \vec{k}$$

$$= \langle 1,-1,1 \rangle .$$

So
$$\left.\begin{aligned} x &= 2+t \\ y &= 1-t \\ z &= t \end{aligned}\right\} \text{parametric eq'ns}$$

$$x-2 = 1-y = z \quad \Big\} \text{ symmetric eqns.}$$

30) Find an eq'n for the plane that contains the line

$$x = 3 + 2t, \quad y = t, \quad z = 8 - t$$

and is parallel to the plane $2x + 4y + 8z = 17$.

Sol'n: This plane must have normal vector $\langle 2, 4, 8 \rangle$ in order to be parallel to the given plane. It also must contain the point $(3, 0, 8)$, which is contained in the line (when $t = 0$).

So the plane has equation

$$\langle x-3, y, z-8 \rangle \cdot \langle 2, 4, 8 \rangle = 0$$

$$\downarrow$$

$$2(x-3) + 4y + 8(z-8) = 0$$

$$\downarrow$$

$$\boxed{2x + 4y + 8z = 70}$$

(62) Find the intersection of the lines

$l_1 : \vec{r} = \langle 1,1,0 \rangle + t\langle 1,-1,2 \rangle$

$l_2 : \vec{r} = \langle 2,0,2 \rangle + s\langle -1,1,0 \rangle$

and an equation for the plane containing them.

Sol'n: At the intersection pt:

$$1+t = 2-s$$
$$1-t = s$$
$$2t = 2$$

So $t=1$ & $s=0$, so $\boxed{(2,0,2) \text{ is the intersection.}}$

The plane containing l_1 & l_2 has normal vector perpendicular to $\langle 1,-1,2 \rangle$ & $\langle -1,1,0 \rangle$.

$$\vec{n} = \langle 1,-1,2 \rangle \times \langle -1,1,0 \rangle$$

$$= \langle -2,-2,0 \rangle$$

so the plane has equation

$$\langle x-2, y, z-2 \rangle \cdot \langle -2,-2,0 \rangle = 0$$

$$\downarrow$$

$$-2(x-2) - 2y = 0$$

$$-2x - 2y = -4$$