M271A Fall 2009 Hw 5

Graded : § 14.3 #22, 46

22 Find the curvature of

$$\vec{r}(t) = t\,\vec{i} + t\,\vec{j} + (1+t^2)\vec{k}$$

Sol'n:

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

$$= \frac{|\langle 1,1,2t\rangle \times \langle 0,0,2\rangle|}{|\langle 1,1,2t\rangle|^3}$$

$$= \frac{|\langle 2,-2,0\rangle|}{\left(\sqrt{1^2+1^2+4t^2}\right)^3} = \boxed{\frac{\sqrt{8}}{(2+4t^2)^{3/2}}}$$

(46) $\langle t, t^2, t^3 \rangle = \vec{r}(t)$.

Find the osculating plane & the normal plane at $(1,1,1)$ $(t=1)$.

Sol'n Osculating plane:

$$\underbrace{\vec{B}(1)} \cdot \langle x-1, y-1, z-1 \rangle = 0$$

or Any vector parallel to $\vec{B}(1)$.

Normal plane:

$$\underbrace{\vec{T}(1)} \cdot \langle x-1, y-1, z-1 \rangle = 0$$

or any vector parallel to $\vec{T}(1)$.

$\vec{r}\,'(1)$ is parallel to $\vec{T}(1)$:

$$\vec{r}\,'(t) = \langle 1, 2t, 3t^2 \rangle$$

$\vec{r}\,'(1) = \langle 1, 2, 3 \rangle$ so the normal plane is

$$\langle 1,2,3 \rangle \cdot \langle x-1, y-1, z-1 \rangle = 0.$$

normal!

Now look at formula for $\vec{T}\,'(t)$ (which points in the direction of $\vec{N}(t)$).

$$\vec{T}\,'(t) = \frac{d}{dt}\left(\frac{1}{|\vec{r}\,'(t)|} \cdot \vec{r}\,'(t) \right)$$

$$= \frac{d}{dt}\left(\frac{1}{|\vec{r}\,'(t)|} \right) \vec{r}\,'(t) + \frac{1}{|\vec{r}\,'(t)|} \cdot \vec{r}\,''(t)$$

Plug in $t=1$: $\vec{T}\,'(1) = \frac{d}{dt}\left(\left(1 + 4t^2 + 9t^4\right)^{-\frac{1}{2}} \right)\Big|_{t=1} \langle 1,2,3 \rangle$

$$+ \left(1 + 4t^2 + 9t^4\right)^{-\frac{1}{2}} \langle 0, 2, 6 \rangle$$

$$\frac{d}{dt}\left((1+4t^2+9t^4)^{-1/2} \right) \Big|_{t=1}$$

$$= \left[-\frac{1}{2}\left(1+4t^2+9t^4\right)^{-3/2} \cdot \left(8t+36t^3\right) \right]_{t=1}$$

$$= -\frac{1}{2} \cdot (14)^{-3/2} \cdot 44 = \frac{-22}{14^{3/2}}$$

So $\vec{T}'(1) = \frac{-22}{14^{3/2}} \langle 1,2,3 \rangle + \frac{1}{14^{1/2}} \langle 0,2,6 \rangle$

$$= \left\langle \frac{-22}{14^{3/2}}, \frac{-44}{14^{3/2}} + \frac{2}{14^{1/2}}, \frac{-66}{14^{3/2}} + \frac{6}{14^{1/2}} \right\rangle$$

$$= \left\langle \frac{-22}{14^{3/2}}, \frac{-16}{14^{3/2}}, \frac{18}{14^{3/2}} \right\rangle$$

$$= \frac{1}{14^{3/2}} \left\langle -22, -16, 18 \right\rangle = \frac{2}{14^{3/2}} \langle -11, -8, 9 \rangle$$

So ~~AN~~ $\cancel{\langle -22,-16,18\rangle}$ $\langle -11,-8,9\rangle$ points in the direction of $\vec{N}(1)$. And $\vec{T}(1)$ is parallel to $\langle 1,2,3\rangle$. So

$$\vec{B}(1) \parallel \underbrace{\langle 1,2,3\rangle \times \langle -11,-8,9\rangle}_{= \langle 42,-42,14\rangle}$$

So the osculating plane is

$$\underbrace{\langle 42,-42,14\rangle \cdot \langle x-1, y-1, z-1\rangle = 0}_{\text{osc. plane}}$$