M 271 A Fall 2009 Hw 6

Graded: § 14.4 # 16, 28, 34

§ 15.1 # 12

16 $\vec{a}(t) = 2\vec{i} + 6t\vec{j} + 12t^2\vec{k}$

$\vec{v}(0) = \vec{i} \qquad \vec{r}(0) = \vec{j} - \vec{k}$.

Find $\vec{v}(t)$ and $\vec{r}(t)$.

Sol'n: $\vec{v}(t) = \int \vec{a}(t)\,dt = (2t + c_1)\vec{i}$

$+ (3t^2 + c_2)\vec{j}$

$+ (4t^3 + c_3)\vec{k}$

$\vec{i} = \vec{v}(0) = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$

$\therefore c_1 = 1$ & $c_2 = 0 = c_3$

$\therefore \boxed{\vec{v}(t) = (2t+1)\vec{i} + 3t^2\vec{j} + 4t^3\vec{k}}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \cdots$

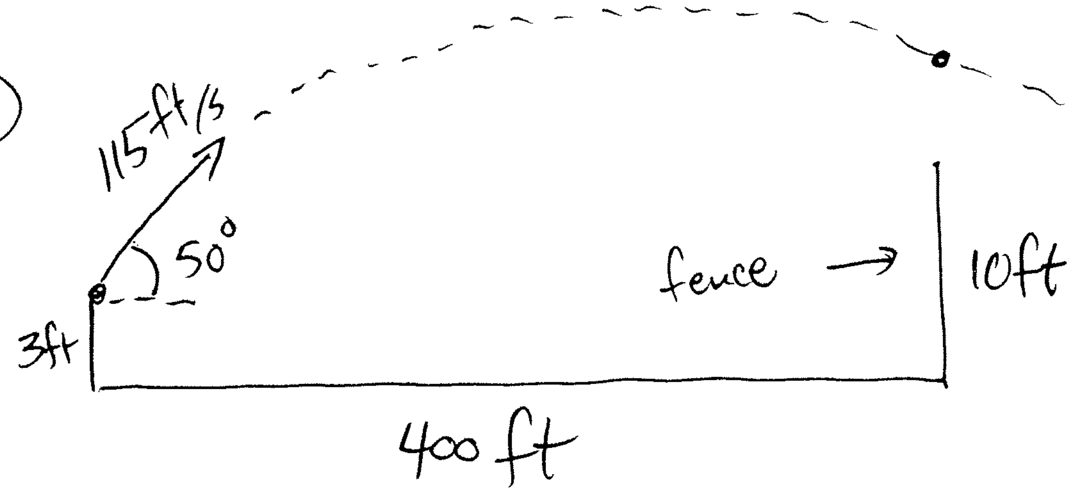
$$= (t^2 + t + c_1)\vec{i} + (t^3 + c_2)\vec{j} + (t^4 + c_3)\vec{k}$$

$$\vec{j} - \vec{k} = \vec{r}(0) = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$$

$$\therefore \quad c_1 = 0, \ c_2 = 1, \ c_3 = -1$$

$$\therefore \quad \boxed{\vec{r}(t) = (t^2 + t)\vec{i} + (t^3 + 1)\vec{j} + (t^4 - 1)\vec{k}}$$

#28



Q: Does the ball clear the fence?

Soln: Find the position function:

$\vec{a}(t) = -32\vec{j} = \langle 0, -32 \rangle$,

$\vec{v}(t) = \langle c_1, -32t + c_2 \rangle$

$\boxed{\begin{array}{l} \vec{v}(0) = \langle 115\cos 50, 115\sin 50 \rangle \\ \vec{r}(0) = \langle 0, 3 \rangle \end{array}}$

$\downarrow$

$\vec{v}(t) = \langle 115\cos 50, 115\sin 50 - 32t \rangle$

$\downarrow$

$\vec{r}(t) = \langle (115\cos 50)t + c_1, (115\sin 50)t - 16t^2 + c_2 \rangle$

$\downarrow$

$\vec{r}(t) = \langle (115\cos 50)t, 3 + (115\sin 50)t - 16t^2 \rangle$

$$(115 \cos 50)\, t = 400$$

$$\downarrow$$

$$t = \frac{400}{115 \cos 50} \approx 5.411 \text{ seconds}$$

when the x-coord of position is 400ft.

plug into the y-coord

$$\downarrow$$

$$3 + (115 \sin 50) \cdot \frac{400}{115 \cos 50} - 16\left(\frac{400}{115 \cos 50}\right)^2$$

$$\approx \boxed{11.219 \text{ ft}}$$

$\leftarrow$ bigger than 10ft, so

A: Yes!

(34) $\vec{r} = \langle 1+t, t^2-2t \rangle$

$$\vec{a} = \underbrace{v'(t)}\vec{T} + \underbrace{v(t)^2 \kappa(t)}\vec{N}$$

We need to calculate these.

$$\vec{v}(t) = \langle 1, 2t-2 \rangle$$

$$|\vec{v}(t)| = v(t) = \sqrt{1+(2t-2)^2} = \sqrt{4t^2-8t+5}$$

$$\vec{T}(t) = \left\langle \frac{1}{\sqrt{4t^2-8t+5}}, \frac{2t-2}{\sqrt{4t^2-8t+5}} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{-\frac{1}{2}\cdot(8t-8)}{(4t^2-8t+5)^{3/2}}, \frac{2\sqrt{4t^2-8t+5} - (2t-2)\frac{1}{2}\frac{(8t-8)}{\sqrt{4t^2-8t+5}}}{4t^2-8t+5} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{4-4t}{(4t^2-8t+5)^{3/2}}, \frac{2}{(4t^2-8t+5)^{3/2}} \right\rangle$$

$$|\vec{T}'(t)| = \sqrt{\frac{(4-4t)^2+4}{(4t^2-8t+5)^3}}$$

$$= \sqrt{\frac{4(4t^2-8t+5)}{(4t^2-8t+5)^3}} = \frac{2}{4t^2-8t+5}$$

so $K(t) = \dfrac{|\vec{T}'|}{|\vec{v}(t)|} = \dfrac{2}{(4t^2-8t+5)^{3/2}}$

$$\boxed{\begin{aligned} v'(t) &= \frac{4t-4}{(4t^2-8t+5)^{1/2}} = \vec{a}_T \\ v^2K &= \frac{2}{\sqrt{4t^2-8t+5}} = \vec{a}_N \end{aligned}}$$

15.1 #12 Find/sketch the domain of

$$f(x,y) = \sqrt{xy}$$

Sol'n:

$$xy \geq 0$$