Math 271 A Fall 2009 Quiz 4

Let P_1 : $2x+3y-z=4$

P_2 : $x-y \quad = 3$

be two planes. (1) Calculate their angle of intersection.

(2) Find parametric equations for their line of intersecti

(3) Is the line $\frac{x-3}{4} = \frac{y+1}{6} = \frac{z}{-2}$ perpendicular to either of P_1 or P_2?

Sol'n: The normal vectors for P_1 and P_2 are

$\vec{n}_1 = \langle 2,3,-1\rangle$ and $\vec{n}_2 = \langle 1,-1,0\rangle$, respectively.

Since the angle between P_1 and P_2 is the angle between $\vec{n}_1$ and $\vec{n}_2$ we compute

$$-1 = 2-3+0 = \vec{n}_1 \cdot \vec{n}_2 = \sqrt{4+9+1}\,\sqrt{1+1}\,\cos\theta$$

so $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{14}\sqrt{2}}\right)$. (1)

To find (2), we need the vector $\vec{n}_1 \times \vec{n}_2$ and a point in the intersection. Set $y=0$ to find where this line meets the xz-plane:

$$\left.\begin{array}{l} 2x + 3\cdot 0 - z = 4 \\ x - 0 = 3 \end{array}\right\} \to \left.\begin{array}{l} 2\cdot 3 + 3\cdot 0 - z = 4 \\ x = 3 \end{array}\right\} \begin{array}{l} z = 2 \\ x = 3 \end{array}$$

So $(3,0,2)$ is a point in the intersection.

$$\vec{n}_1 \times \vec{n}_2 : \quad \left.\begin{array}{cccccc} i & j & k & i & j & k \\ 2 & 3 & -1 & 2 & 3 & -1 \\ 1 & -1 & 0 & 1 & -1 & 0 \end{array}\right\} \to \langle -1, -1, -5 \rangle$$

So $x = 3 - t$, $y = 0 - t$, $z = 2 - 5t$ is the line.

For 3, we observe that this line has the direction vector $\langle 4, 6, -2 \rangle = 2\langle 2, 3, -1 \rangle = 2\vec{n}_1$, so the line is parallel to $\vec{n}_1$ and therefore $\perp$ to P_1.