Math 271A | Multivariable Calculus I
Fall 2009 | Quiz 4
Instructor: Shawn Rafalski | **Write your name on this quiz**

Consider the vector function $\mathbf{r}(t) = \langle e^{-t}, 2\sin t, t^2\rangle$, and let $C$ be the spacecurve determined by this $\mathbf{r}(t)$.

1. Calculate the unit tangent vector $\mathbf{T}$ of $C$ at the point $(e^{-\pi}, 0, \pi^2)$.

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -e^{-t}, 2\cos t, 2t\rangle}{|\langle -e^{-t}, 2\cos t, 2t\rangle|}$$

At $(e^{-\pi}, 0, \pi^2)$, $t = \pi$ so we want $\vec{T}(\pi)$:

$$\boxed{\vec{T}(\pi) = \frac{\langle -e^{-\pi}, -2, 2\pi\rangle}{\sqrt{e^{-2\pi} + 4 + 4\pi^2}}}.$$

2. Calculate parametric equations for the tangent line to $C$ at the point $(e^{-\pi}, 0, \pi^2)$.

point: $(e^{-\pi}, 0, \pi^2)$.

vector: $\vec{r}\,'(\pi) = \langle -e^{-\pi}, -2, 2\pi\rangle$

so
$$\boxed{\begin{aligned} x &= e^{-\pi} - t e^{-\pi} \\ y &= 0 - 2t \\ z &= \pi^2 + 2\pi t \end{aligned}}$$