Math 172A
Spring 2010
Instructor: Shawn Rafalski

Integral Calculus
Exam 2

Write your name on this exam right now. Your work on this exam is to be your work alone. No calculators allowed. You have 60 minutes to finish. Explain your answers clearly, and *show your work*. This exam has 9 pages, and the questions are worth a total of 100 points (not including bonus points). Only work on the bonus questions **after** you have tried to do all the regular questions. Don't forget to breathe regularly, and good luck!!

Solution

Begin working on the next page.

1. (5 points each) Compute the derivatives of the following functions. You do not have to simplify your answers.

(a) $y = \ln(x^3 \arctan x)$

$$\frac{3x^2 \arctan x + x^3\left(\frac{1}{1+x^2}\right)}{x^3 \arctan x}$$

(b) $y = e^{3e^x}$

$$e^{3e^x} \cdot \left(3e^x\right)$$

(c) $y = \sin^{-1}(\sqrt{x})$

$$\frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \left(\frac{1}{2}x^{-1/2}\right)$$

2. Evaluate the following integrals, or show that they are divergent. You do not have to simplify your answers.

   (a) (6 points) $\displaystyle\int_0^1 \arcsin x \, dx$

$u = \arcsin x \qquad v = x$

$du = \dfrac{1}{\sqrt{1-x^2}} dx \qquad dv = dx$

$= x \arcsin x \Big|_0^1 - \displaystyle\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$

$= \boxed{\pi/2 - 1}$

   (b) (6 points) $\displaystyle\int \frac{\tan^{-1} x}{1+x^2} \, dx$

$u = \tan^{-1} x$

$du = \dfrac{1}{x^2+1} dx$

$= \boxed{\frac{1}{2}(\tan^{-1} x)^2 + C}$

(c) (9 points) $\int_{-1}^{2} \frac{dx}{x^2}$

$$\| $$

$$\int_{-1}^{0} \frac{dx}{x^2} + \int_{0}^{2} \frac{dx}{x^2}$$

both improper integrals diverge.

(d) (9 points) $\int_{0}^{\infty} xe^{-x}\, dx$

$u = x \qquad v = -e^{-x}$

$du = dx \quad dv = e^{-x}\, dx$

$$\lim_{t\to\infty}\left(\int_0^t xe^{-x}\,dx\right) = \lim_{t\to\infty}\left\{ -xe^{-x}\Big|_0^t + \int_0^t e^{-x}\,dx \right\}$$

$$\| $$

$$\boxed{1}$$

3. (5 points each) Simplify the following expressions.

(a) $e^{2\ln 3}$

$\Vert$

$e^{\ln(3^2)} = 3^2 = 9$

(b) $\ln(\ln(e^{e^7}))$

$\Vert$

$\ln(e^7) = 7$

(c) $\sin^{-1}(\tan(\pi/4))$ $= y$

$\sin y = \tan\frac{\pi}{4} = 1$

$\Downarrow$

$y = \pi/2$

(d) $\cos(\arcsin(\sqrt{3}/2))$

$$\theta = \sin^{-1}\left(\sqrt{3}/2\right) \longrightarrow \sin\theta = \sqrt{3}/2$$

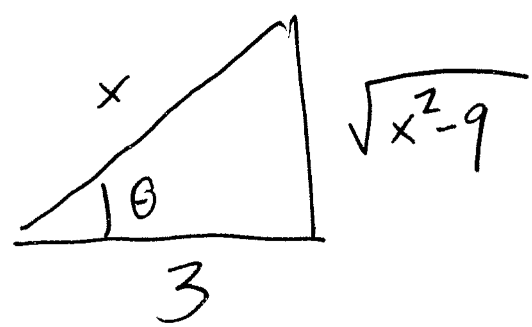
$$\downarrow$$

$$\cos \pi/3 = \boxed{\frac{1}{2}} \longleftarrow \theta = \pi/3$$

(e) $\tan(\cos^{-1}(3/x))$

$$\theta = \cos^{-1}\left(3/x\right) \longrightarrow \cos\theta = 3/x$$

$$\tan\theta = \boxed{\frac{\sqrt{x^2-9}}{3}} \longleftarrow$$



4. (6 points each) Evaluate the following limits, if possible.

(a) $\lim_{x\to 1} \frac{x^9 - 1}{x^5 - 1}$ $= \lim_{x \to 1} \frac{9x^8}{5x^4} = 9/5$

(b) $\lim_{x\to -\infty} \arctan x$ $= -\pi/2$

(c) $\lim_{x\to\infty} \frac{\ln x}{e^x}$ $= \lim_{x\to\infty} \frac{1/x}{e^x} = 0.$

(d) $\lim_{x\to\infty} x^{1/x}$

$\boxed{y = x^{1/x}} \to \ln y = \frac{1}{x}\ln x$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \underbrace{\frac{1}{x}\ln x}_{\text{"}\frac{\infty}{\infty}\text{"}} = \lim_{x\to\infty} \frac{1/x}{1} = 0$$

So $\lim_{x\to\infty} y = e^0 = 1$

(e) $\lim_{x\to 0} \ln(e^x) = \lim_{x\to 0} x = 0$

5. **(Bonus 1 point each!)**

   (a) What was the name of the computer in the Human vs. Computer game from Dr. King's chess talk?

   (b) What was the name of the human in the Human vs. Computer game from Dr. King's chess talk?

   (c) Name one of the three brands of clothing that Dr. King thinks the typical college student wears.